Adaptive Beamforming Algorithm based on a Simulated Kalman Filter

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***Abstract* - A new population-based metaheuristic optimization algorithm named Simulated Kalman Filter (SKF) is proposed as an adaptive beamforming algorithm for adaptive array antenna. SKF optimization algorithm is inspired by the estimation capabilities of Kalman Filter. Each agent in the population of SKF acts as one Kalman Filter where it finds the solution using standard Kalman Filter framework. SKF consists of simulated measurement process and a best-so-far solution as a reference. SKF estimates the weights of individual elements in an array which maximizes the signal to interference plus noise ratio (SINR). SKF is also compared with Adaptive Mutated Boolean Particle Swarm Optimization (AMBPSO) from existing work and is proven to be better.**

Keywords - Adaptive Beamforming; Simulated Kalman Filter

I. Introduction

Signal environments such as wireless cellular communication system involves time-varying signal propagation environment where the user and interferers move around with time. Adaptive beamforming is used to continuously adapt with the changing electromagnetic environment by continuously adjusting the weights of individual elements in an array. In adaptive beamforming techniques, the main beam must be pointed towards the direction of the desired signal and nulling the interference at the same time.

To date, there are a number of optimization algorithms that were applied in adaptive beamforming [1]–[13]. These algorithms are used to find the optimum weights so as to steer the main beam towards the signal of interest (SOI) and null the interference to maximize the signal to interference plus noise ratio (SINR) value. In this paper, a new metaheuristic optimization algorithm named Simulated Kalman Filter (SKF) [14] is proposed for adaptive beamforming application. The SKF, introduced by Ibrahim *et at.*, was inspired by the estimation capabilities of Kalman Filter and has already been applied in various optimization problem [15]–[19]. SKF is used to estimate weights of individual elements in an array which gives maximum signal to interference plus noise ratio (SINR) value.

II. System Model

Assuming an array antenna of M elements and N number of interfering signal with signal of interest (SOI) of kth time sample, $s(k)$, arriving at angle θ_0, and signal not of interest (SNOI), $i_1(k)$, $i_2(k)$, $i_3(k)$, …, $i_{N-1}(k)$, $i_N(k)$, arriving at angle θ_1, θ_2, θ_3, …, θ_{N-1}, θ_N, as shown in Fig. 1 [20].

The array output, $y(k)$ can be represented by

$$y(k) = \bar{w}^H \cdot \bar{x}(k) \tag{1}$$

where w stands for weights for individual elements, H for Hermitian transpose and $\bar{x}(k)$ stands for the signal vector. The signal vector, $\bar{x}(k)$ can be formulated as

$$\begin{aligned}\bar{x}(k) &= \bar{a}_0 s(k) + [\bar{a}_1 \bar{a}_2 \cdots \bar{a}_N] \cdot \begin{bmatrix} i_1(k) \\ i_2(k) \\ \vdots \\ i_N(k) \end{bmatrix} + n(k) \\ &= \bar{x}_s(k) + \bar{x}_i(k) + \bar{n}(k)\end{aligned} \tag{2}$$

where $\bar{a}_i$ stands for the M-element array steering vector for θ_i direction of arrival, $\bar{x}_s(k)$ is the desired signal vector, $\bar{x}_i(k)$ is the interfering signal vector and $\bar{n}(k)$ is the noise.

The total array output, $y(k)$ is expanded as

$$\begin{aligned}y(k) &= \bar{w}^H \cdot [\bar{x}_s(k) + \bar{x}_i(k) + \bar{n}(k)] \\ &= \bar{w}^H \cdot [\bar{x}_s(k) + \bar{u}(k)]\end{aligned} \tag{3}$$

where the undesired signal, $\bar{u}(k)$, can be formulated as

$$\bar{u}(k) = \bar{x}_i(k) + \bar{n}(k) \tag{4}$$

Next, the array correlation matrices are calculated for both desired signal, $\bar{R}_{ss}$ and undesired signal $\bar{R}_{uu}$. The weighted array output power for desired signal is given as follows

$$\sigma_s^2 = E[|\bar{w}^H \cdot \bar{x}_s|^2] = \bar{w}^H \cdot \bar{R}_{ss} \cdot \bar{w} \tag{5}$$

where the signal correlation matrix, $\bar{R}_{ss}$, can be formulated as

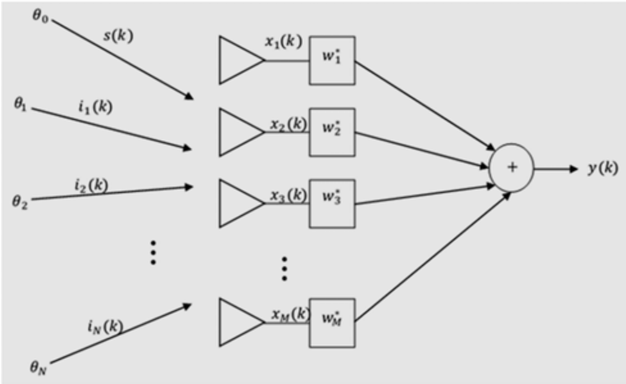


Fig. 1: Array model.

$$\bar{R}_{ss} = E[\bar{x}_s \bar{x}_s^H] \tag{6}$$

For the undesired signal, the weighted array output power, σ_u^2 is

$$\sigma_u^2 = E[|\bar{w}^H \cdot \bar{u}|^2] = \bar{w}^H \cdot \bar{R}_{uu} \cdot \bar{w} \tag{7}$$

In equation (7), the $\bar{R}_{uu}$ is formulated as

$$\bar{R}_{uu} = \bar{R}_{ii} + \bar{R}_{nn} \tag{8}$$

where $\bar{R}_{ii}$ denotes the denotes the interference correlation matrix and $\bar{R}_{nn}$ denotes noise correlation matrix.

The signal to interference plus noise ratio, SINR, can be formulated as

$$SINR = \frac{\sigma_s^2}{\sigma_u^2} = \frac{\bar{w}^H \cdot \bar{R}_{ss} \cdot \bar{w}}{\bar{w}^H \cdot \bar{R}_{uu} \cdot \bar{w}} \tag{9}$$

III. Simulated Kalman Filter

Based on the capability of Kalman Filter in state estimation, each agent in SKF able to improve its estimation of the optimum. In SKF algorithm, each agent acts as an individual Kalman Filter. Consider there are N agents and t indicates the iteration number, the estimated solution of an optimization problem of the i^{th} agent at a time t, $X_i(t)$, is defined as:

$$X_i(t) = \{x_i^1(t), x_i^2(t), \cdots, x_i^d(t), \cdots, x_i^D(t)\} \tag{10}$$

where $x_i^d(t)$ represents the estimated state of the i^{th} agent in the d^{th} dimension and D is defined as the maximum number of dimensions. In an iteration t, a number of agents are involved in the calculation of fitness, and then, an agent with the best fitness, $X_{best}(t)$, is identified. The SKF algorithm also requires a simulated measurement process, which is led by a true value, X_{true}. The X_{true} represents the best solution-so-far and it is updated when a better solution than X_{true} is found.

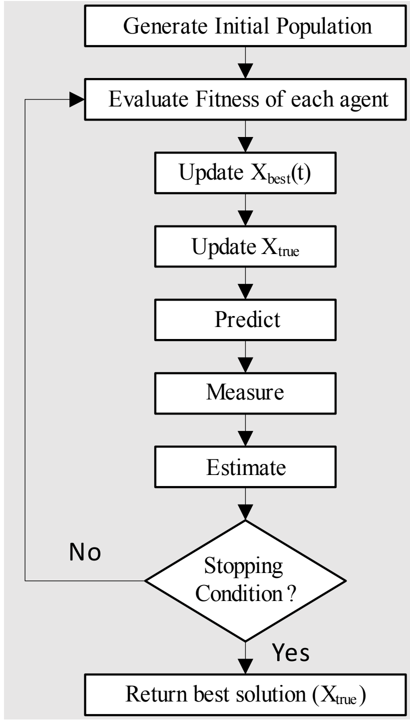


Fig. 2: Flowchart of SKF

The SKF algorithm is illustrated in Fig. 2. The SKF algorithm starts with random initialization of its agents' estimated state, $X(0)$, within the search space. Besides the initial state estimate, the initial value of error covariance estimate, $P(0)$, the process noise, Q, and the measurement noise, R, are also defined during initialization stage. The maximum number of iterations, $tMax$, is also initialized.

The iteration begins with the fitness calculation of the i^{th} agent, $fit_i(X(t))$. Then, the $X_{best}(t)$ is updated according to the type of problem. In minimization problem,

$$X_{best}(t) = min_{i\in 1,2,\ldots,N} fit_i(X(t)) \tag{11}$$

whereas, for maximization problem,

$$X_{best}(t) = max_{i\in 1,2,\ldots,N} fit_i(X(t)) \tag{12}$$

After that, the true value, X_{true}, is updated. Note that the X_{true} represents the best solution-so-far. Thus, X_{true} is updated if a better solution ($X_{best}(t) < X_{true}$ for minimization problem, or $X_{best}(t) > X_{true}$ for maximization problem) is found.

SKF search strategy contains three simple steps; predict-measure-estimate. Thus, the two sets of Kalman equations are adopted in SKF. During prediction, the time-update equations are used to obtain the a priori estimates for the next time step. After the measurement process, measurement-update equations are used to obtain the improved a posteriori estimates.

TABLE I. PARAMETER FOR BENCHMARK

Desired Signal (Theta, θ)	30°
Interference Signal (Theta, θ_i)	$-70°, -40°, -30°, -10°, 0°, 10°, 50°, 70°$
Iteration (SKF)	10000
Agents (SKF)	100
Number of Runs	100
Geometry	Linear
Number of Elements	10
Distance	0.5λ

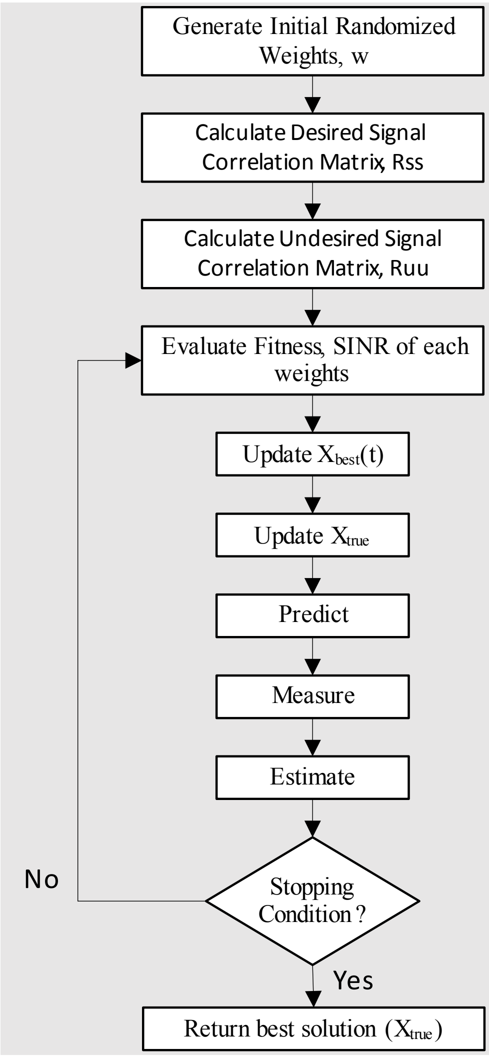


Fig. 3: The flowchart of SKF for adaptive beamforming.

In the prediction step, the following time-update equations:

$$x_i(t|t+1) = x_i(t) \tag{13}$$

$$P(t|t+1) = P(t) + Q \tag{14}$$

are employed to make a prediction of the state and error covariance estimates given the prior estimates. These estimates are called the *a priori* estimates. The next step is measurement. Measurements act as feedback to estimation process. Measurement of each individual agent is simulated based on the following equation:

$$z_i(t) = x_i(t|t+1) + sin(rand \times 2\pi) \times |x_i(t|t+1) - X_{true}| \tag{15}$$

where the $rand$ is a uniformly distributed random number in the range of [0,1]. Given the predicted state estimate, $x_i(t|t+1)$, measurement may take any random value from the predicted state estimate, $x_i(t|t+1)$, to the true value, X_{true}. A random element, $rand$, in $\sin(rand \times 2\pi)$ term is important to induce stochastic behavior in SKF.

The final step is the estimation. During this step, Kalman gain, $K(t)$, is computed as follows:

$$K(t) = \frac{P(t|t+1)}{P(t|t+1) + R} \tag{16}$$

Then, the measurement-update equations are used to improve the *a posteriori* estimates from the *a priori* estimates by making use of the measurement.

$$x_i(t+1) = x_i(t|t+1) + K(t) \times \left(z_i(t) - x_i(t|t+1)\right) \tag{17}$$

$$P(t+1) = (1 - K(t)) \times P(t|t+1) \tag{18}$$

Using the measured position as feedback and influenced by the Kalman gain value, $K(t)$, each agent updates an estimate of the optimum for that corresponding iteration. The next iteration is executed until the maximum number of iterations, $tMax$ is reached.

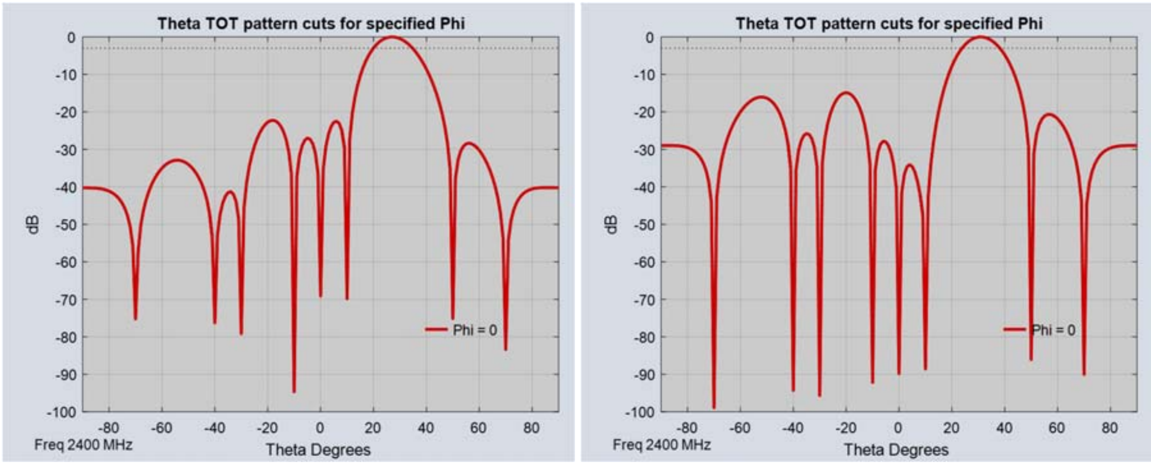
IV. EXPERIMENTAL SETUP

SKF is compared with existing work, AMBPSO by Z.D. Zaharis and T.V. Yioultsis [6] with parameters as shown in Table I. The SKF is used to find the optimum weights by maximizing the SINR fitness function. Figure 3 shows the flowchart of application of SKF in adaptive beamforming.

The Wilcoxon signed ranked test is also performed to determine the level of significance between AMBPSO and SKF. The significance level, α, is set to 0.05.

V. SIMULATION RESULTS AND DISCUSSION

Fig. 4 shows the radiation pattern comparison for AMBPSO (replotted with element weights [6]) and SKF for SNR value of $30dB$. From Fig. 4, SKF able to produce much deeper nulls compared to AMBPSO but AMBPSO is able to produce much lower sidelobe level (SLL) compared with. Table II shows the benchmark result between AMBPSO and SKF.



-A. –B.

FIG. 6: RADIATION PATTERN: (A) AMBPSO, (B) SKF

TABLE II. SINR VALUE COMPARISON

SNR(dB)	AMBPSO				SKF			
	Best	Worst	Mean	STD	Best	Worst	Mean	STD
-20	-10.0522	-10.0548	-10.0523	0.0004	-10.0522	-10.0549	-10.0523	0.0004
-15	-5.1395	-5.1512	-5.1399	0.0020	-5.1395	-5.1464	-5.1398	0.0011
-10	-0.2975	-0.3692	-0.2998	0.0098	-0.2975	-0.3028	-0.2977	0.0007
-5	4.5321	4.3422	4.5269	0.0218	4.5321	4.3828	4.5295	0.0175
0	9.4241	8.5481	9.3749	0.1643	9.4241	9.3237	9.4208	0.0142
5	14.3768	12.0647	14.2676	0.3628	14.3768	14.1437	14.3631	0.0306
10	19.3598	15.1371	19.2810	0.4463	19.3590	18.8208	19.2736	0.0954
15	24.3542	16.5370	24.1008	1.0643	24.3535	23.0034	24.1049	0.2935
20	29.3509	17.2416	29.0332	1.2290	29.3469	28.0453	29.0012	0.3236
25	34.3515	22.4314	33.6680	1.4163	34.3504	32.7718	33.9332	0.3790
30	39.3341	30.2715	38.7648	0.8722	39.3477	37.5436	38.9249	0.3946
35	44.3440	32.6393	43.1564	1.5287	44.3510	41.8109	43.9153	0.4384
40	49.3461	36.6898	48.1781	1.2409	49.3456	46.3051	48.9181	0.4529
45	54.3358	43.2386	52.5134	1.5788	54.3504	51.9201	53.8808	0.4746
50	59.3317	47.6499	58.3221	1.7439	59.3355	55.8211	58.8351	0.6189
55	64.3458	52.7630	63.0660	1.6119	64.3468	62.0234	63.8755	0.5364
60	69.3468	56.3379	67.5858	1.8369	69.3396	67.7525	68.9202	0.4239

Neutral
Good

From Table II, SKF gives better SINR mean value compared with AMBPSO with different SNR values. SKF is also proven to be more stable than AMBPSO with lower standard deviation value.

For statistical analysis, Wilcoxon Signed Ranked Test is performed. The sum of positive ranks, R^+ and the sum of negative rank, R^- is as shown in Table III.

TABLE III. WILCOXON SIGNED RANKED TEST

	R^-.	R^+
Total	14	139

The test statistic, T is the smaller value of either sum of positive ranks, R^+ or the sum of negative rank, R^-. From Table III, the test statistic, T is equal to 14. For $\alpha = 0.05$ and $n = 17$, the critical value, T_0 in Wilcoxon Signed Rank Test is equal to 35. The test statistic, T less than the critical value, T_0, therefore, the results is proven to be significant.

VI. CONCLUSION

This paper presents the first application of Simulated Kalman Filter (SKF) for adaptive beamforming application. SKF is proven to be better than AMBPSO in mean SINR values almost every SNR values and is also consistent in maintaining its maximum SINR value for every SNR values. Results obtained using SKF is statistically proven to be significant when comparing with AMBPSO using Wilcoxon Signed Ranked Test.

ACKNOWLEDGEMENT

This research is supported by Fundamental Research Grant Scheme (RDU160105) awarded by Ministry of Higher Education Malaysia to Universiti Malaysia Pahang.

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