Thermal Performance of Heat Conduction in a Square Plate

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***Abstract* – In this paper, a numerical solution of heat conduction in a two-dimensional plate at different tilt angles and convection heat transfer coefficients is presented. The finite difference method is used to solve the steady state differential equation of heat conduction in a plate. A set of algebraic equations are formulated for interior nodes and surface faces of the plate. Temperature distribution and heat flux is presented for different tilt angle ranges from 15º to 90º. The results show the temperature distribution at all points in the plate. It is also observed that the heat flux decreases as the distance increases. In addition, heat flux is predicted for different heat transfer coefficients. It is found that the heat flux increases as the heat transfer coefficient increases.**

Keywords – numerical, heat conduction, temperature distribution, heat flux, finite difference

I. INTRODUCTION

The problem of heat conduction in a plate has been studied by many researchers. Crittenden and Cole [1] determined the temperature and heat flux in a rectangular parallelepiped. An algorithm is presented to obtain the results at different boundary conditions. A computation of temperature under steady state in multi-dimensional multi-layer bodies was presented by Haji et al. [2]. In addition, Sun and Wichman [3] presented a theoretical solution to heat conduction in a composite slab. Eigen function expansion method was used to provide exact solution. Luttich et al. [4] specified different boundary conditions at two dimensional plate and obtained results for heat flux in the plate. Beck and Cole [5] used separation of variables method to solve a heat conduction problem in a two dimensional rectangle. Kidawa [6] developed analytical solution to determine the temperature distribution in a plate subjected to moving heat source. Exact solution to the problem using Green's function method was obtained. Beck et al. [7] studied steady state heat conduction in a rectangular plate under different boundary conditions. Peng et al. [8] developed a heat transfer models for a photovoltaic (PV) wall mounted on multi-layer wall to predict the temperature distribution PV panel. Al-Kouz et al. [9] studied numerically a two dimensional convection heat transfer in a cavity with two attached solid fins. Temperature variation in the cavity and along the fins was presented.

In this study, numerical solution to the heat conduction in a square plate is presented. Steady state heat conduction differential equation is solved using finite difference method at specified boundary conditions at the plate. Temperature distribution and heat flux are predicted at different inclined angle.

II. MATHEMATICAL MODEL

A two-dimensional square plate that is 1 m on a side made of aluminum having a thermal conductivity of 204 W/m.K is considered in this study as shown in Figure 1. The plate is subjected to a uniform temperature of 70 ^{0}C at the bottom. The top and left side of the plate is insulated, while the right side is exposed to convection with air at 20 ^{0}C and heat transfer coefficient varied from 10 - 40 W/m^2.K. The plate tilt angle θ is defined as the angle between the plate direction and the horizontal plane which ranges from 15^0 to 90^0.

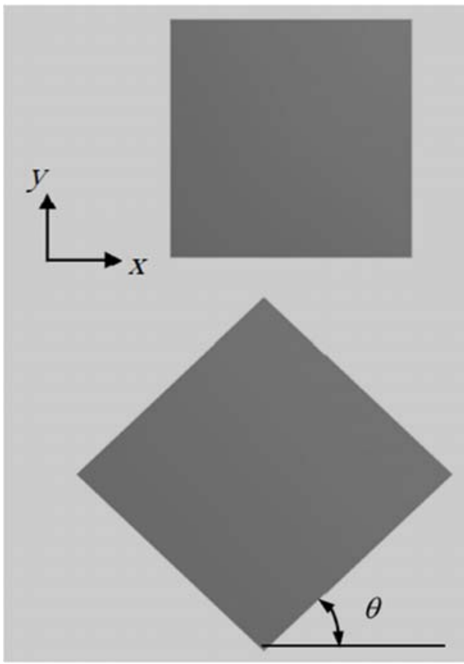

Figure 1. Two-dimensional square plate.

Under steady state, constant thermal conductivity, and two-dimensional conditions, the differential equation of heat conduction is expressed by:

$$\frac{\partial^2 T}{\partial x^2} + \frac{\partial^2 T}{\partial y^2} = 0 \quad (1)$$

where T is the surface temperature of the plate in x and y direction.

In order to predict the temperature distribution in the plate, equation (1) solved using finite difference method to be reduced to an algebraic equation. For interior nodes that equidistant from its four neighboring nodes, the algebraic equation is given as:

$$T_{i,j+1} + T_{i,j-1} + T_{i+1,j} + T_{i-1,j} - 4T_{i,j} = 0 \quad (2)$$

In case of insulated condition at the left side of the plate, the equation is:

$$T_{i,j+1} + T_{i,j-1} + 2T_{i+1,j} - 4T_{i,j} = 0 \quad (3)$$

For the insulated top side, the equation is:

$$2T_{i,j-1} + T_{i+1,j} + T_{i-1,j} - 4T_{i,j} = 0 \quad (4)$$

The finite difference equation of the right surface with convection is given by:

$$\left(2T_{i-1,j} + T_{i,j+1} + T_{i,j-1}\right) + \frac{2h\Delta x}{k} T_\infty - 2\left(\frac{h\Delta x}{k} + 2\right) T_{i,j} = 0 \quad (5)$$

where h is the heat transfer coefficient, k is the thermal conductivity of the plate, T_∞ is the ambient air temperature, and Δx is the distance between two neighboring nodes.

The heat flux out of the plate is determined by computing convection heat transfer from the right surface of the plate. That is:

$$q'' = h(T_{i,j} - T_\infty) \quad (6)$$

where q'' is the convection heat flux.

III. RESULTS AND DISCUSSIONS

A numerical study is performed using finite difference method to predict the temperature distribution and heat flux on aluminum plate. The plate is discretized into 100 elements with 121 nodes as shown in Figure 2.

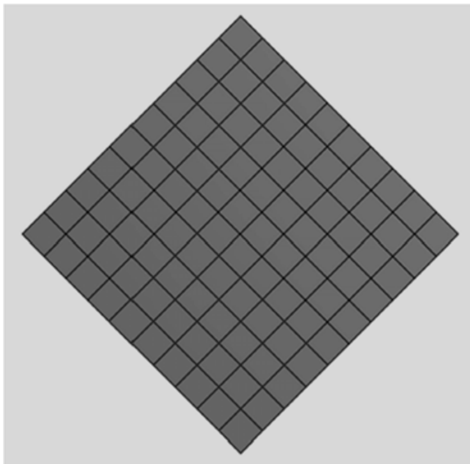

Figure 2. Computational model.

The temperature distribution in the square plate surface is obtained at different tilt angle θ. Figure 3 presents the temperature contour at $\theta = 15^o, 45^o, 60^o, \text{and } 90^o$. It can be seen from the figures the temperature distributed from the hot surface at the bottom to the right surface. This is due to the heat transfer by conduction from high temperature to low temperature on the plate. The figures also show that the effect of tilt angle on temperature distribution. It is observed that when the plate rotates by angle 15° and 60°, the temperature variations are the same as that at 90°. Whereas, when the plate rotates by 45°, the temperature variations are reduced.

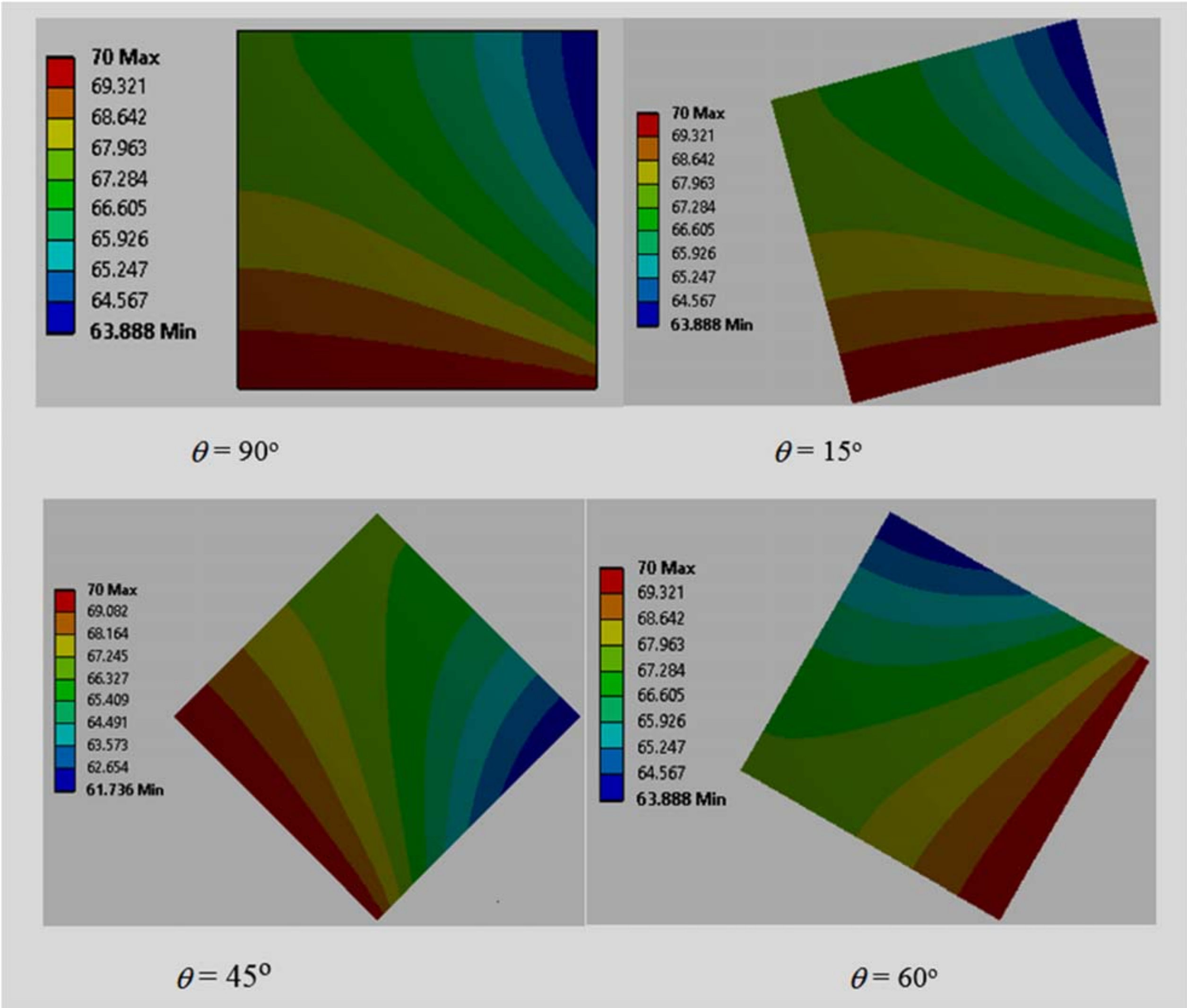

Figure 3. Temperature distribution contour at different θ s.

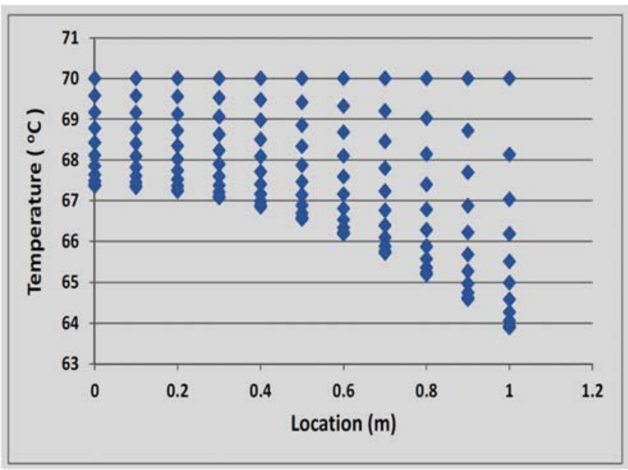

Figure 4. Variations of temperature in the plate.

Variation of temperature in the plate at different location is plotted in Figure 4. The figure shows that the temperature decreases from 70 ^{0}C to about 64 ^{0}C. This is due to the heat transfer by convection to the ambient air.

In Figure 5, variations of the heat flux from the plate surface to the ambient air are predicted at different tilt angle. The figure shows that the heat flux for 15^o, 60^o, and 90^o are identical. However, when the plate rotate by 45^o, the ability of heat transfer to the ambient air is reduced. The figure also shows that the heat flux decreases with the distance from bottom to top surface for all tilt angles.

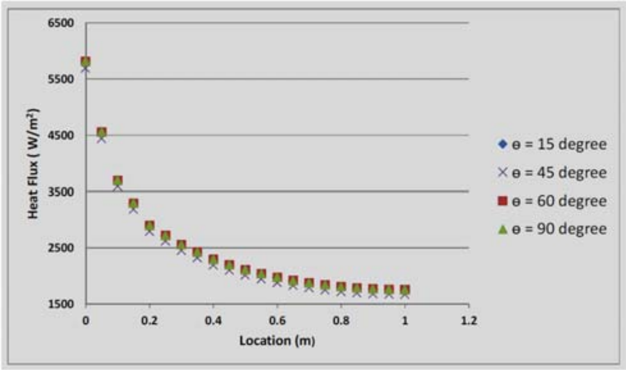

Figure 5. Variations of heat flux with different tilt angles at the convection surface.

Figure 6 shows the predicted heat flux at different heat transfer coefficient. It can be seen that the heat flux increases as the heat transfer coefficient increases from 10 to 40 $W/m^2.K$. The maximum heat loss from the plate can be achieved at heat transfer coefficient of 40 $W/m^2.K$.

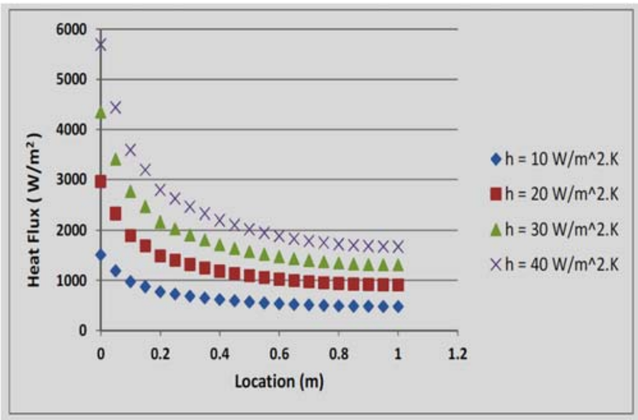

Figure 6. Variations of heat flux with different heat transfer coefficient.

IV. CONCLUSION

In this paper, numerical investigation of thermal performance in square aluminum plate was proposed. The finite different method was used to solve the differential equation of heat conduction in two-dimensional plate. The method described the temperature distribution and heat flux at different tilt angle and heat transfer coefficient. It was found that the temperature decreases along the plate due to heat convection to the ambient air for all tilt angles. Furthermore, the heat flux was increased by increasing the convection heat transfer coefficient. Further study is needed to investigate experimentally the thermal performance of the plate.

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