Hybrid Techniques for Reduction of Linear Time-Invariant Systems

Shilpi Lavania
Dr A P J Abdul Kalam Technical University
Lucknow, India.
shilpi.lavania.vlsi@gmail.com

Deepak Nagaria
Bundelkhand Institute of Engineering & Technology
Jhansi, India.

***Abstract* - Model order reduction (MOR) has an unprecedented impact on modeling of complex control system problems. This paper strives to propose three novel combinational diminution methods for order reduction of complex high order linear time-invariant (LTI) single input-single output (SISO) systems. In each method described, the diminutive approximants for denominator are derived using modified Padé which includes matching of both markov parameters and time moments, whereas numerator in each method is derived using different MOR techniques. These three methods for the evaluation of numerator approximants are Factor division algorithm, Cauer II form and Cauer III form. Further these hybrid methods are compared with the existing model order reduction techniques to analyze their performance. Performance is assessed in respect of a performance index (PI) known as Integral square error (ISE). All suggested methods guarantee stability of the obtained lower order system provided that the original high order system is stable.**

Keywords - Complex systems, Model Order Reduction (MOR), Single Input Single Output (SISO) systems, Integral Square Error (ISE), Padé Approximation, Performance Index (PI) .

I. INTRODUCTION

MOR is essential in system foundation and development. Field of model order reduction is pioneering, sophisticated, substantial and impelling. Replacement of higher order model with its lower order approximants provides ease of simulation, implementation and development of prototype devices i.e. implementation of controller in real time. Large scale system induces large simulation time and ample parallel computing which makes it costly and vigorous to be implemented in real time. MOR provides reduced order approximants which are as much efficient as that of the original system of higher degree with an additional benefit of small dimensions. This advantage has leaded the field model order reduction to an extensively inventive area of research [1-4].

Enormous researches have been carried out to overcome the dimensionality problem of higher order system (HOS) aeons. An extensive range of techniques for MOR have been developed in recent years. MOR techniques started when Davison proposed a simplification method to reduce linear system dynamic system in 1966[2] and 1967 Chidambara proposed modification to the Davison method [5-7]. Later numerous techniques were developed and classified in two domains I) Time domain and II) Frequency domain. Various methods [9] i.e. Modal method Singular method, Optimal Solution etc. have been suggested in time domain and Routh Approximation, Padé approximation, Continued Fraction, Pole clustering etc. [8-9] in frequency domain [3]. Factor division algorithm was first introduced by Lucas in 1983 [10] and later its variant appeared in 1986 [6-7]. To deduce the reduced order approximants of complex high order SISO system, Chen and Shieh proposed continued fraction expansion (CFE) technique for the very first time in 1968. Several CFE techniques including Cauer I form, Cauer II form, Cauer III form and Cauer modified form were proposed as variants of CFE methods [10-12]. Padé approximation has emerged as a powerful tool in developing reduced order approximants of higher order system. It was first proposed by Padé [13].The reduction technique involved in Padé approximation is founded on matching various terms which appear in the series expansion about s=0. Various benefits associated with Padé approximation are that diverse parameter values of reduced order system i.e. fitting time moments, effortless computation, and steady state are same as that of the original system [7].

This paper endeavors to propose three novel composite methods to obtain reduced order approximants for HOS. For obtaining reduced order approximants of denominator polynomial, a modified Padé technique which involves matching of both markov's parameters and time moments is used. Whereas, Factor division algorithm, Cauer II form and Cauer III form are employed to find reduced order approximants for numerator polynomial. Numerical examples are solved by applying suggested methods. A detailed comparison which is founded on a PI i.e. ISE is also manifested in this work.

II. PROBLEM FORMULATION

An n^{th} order linear time invariant (LTI) single input-single output (SISO) HOS is considered as follows:

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$$G_n(s) = \frac{a_1 s^{n-1} + a_2 s^{n-2} + ... + a_n}{s^n + b_1 s^{n-1} + ... + b_n} \quad (1)$$

Where, $a_1, a_2, ...a_n$ and $b_1, b_2, ...b_n$ are scalar constants. The aspiration is to realize an r^{th} order model where (r<n), which comprises $p_1', p_2', ...p_r'$ and $q_1', q_2', ...q_r'$ as scalar constants and is represented as:

$$G_r(s) = \frac{p_1' s^{r-1} + p_2' s^{r-2} + ... + p_r'}{s^r + q_1' s^{r-1} + ... + q_r'} \quad (2)$$

III. MODEL ORDER REDUCTION METHODS

This section is subdivided in to two subsections. The first section defines the technique utilized to obtain reduced order approximants for the denominator polynomial. The second section specifies three different techniques to acquire reduced order approximants for numerator polynomial.

A. Method to Obtain Reduced Order Denominator

A1. Padé Approximation (Moment Matching and Markov Parameters)

For an n^{th} order system [9,14]:

$$G_n(s) = \frac{a_1 s^{n-1} + a_2 s^{n-2} + ... + a_n}{s^n + b_1 s^{n-1} + ... + b_n} \quad (3)$$

On expanding (3) about $s = 0$ and $s = \infty$ will produce (4) and (5) respectively as follows:

$$= t_0 + t_1 s + t_2 s^2 + ... \quad (4)$$

$$= M_1 s^{-1} + M_2 s^{-2} + ... \quad (5)$$

$$G_r(s) = \frac{p_1' s^{n-1} + p_2' s^{n-2} + ... + p_r'}{s^n + q_1' s^{n-1} + ... + q_r'} \quad (6)$$

On expanding (3) about $s = 0$ and $s = \infty$ will produce (7a) and (7b) respectively as follows:

$$= t'_0 + t_1' s + t_2' s^2 + ... \quad (7a)$$

$$= M_1' s^{-1} + M_2' s^{-2} + ... \quad (7b)$$

Using following equations it is simple to validate following equations [9]:

For i=1;

$$t_r' = \frac{p_r'}{q_r'} \quad (8)$$

For i=2,3,4……..

$$t_i' = p'_{r+1-i} + \sum_{j}^{i-1} (t'_i q'_{r+j-i}) q_r'^{-1} \quad (9)$$

and $p_i' = 0$ for $i \le 0; q_0' = 1; q_i' = 0$ for $i \le -1$

$$M_i' = p_i' \text{ for } i = 1, \; M_i' = p_i' - \sum_{j-1}^{i} M_i' q_{i-j}' \quad i = 1,2,3.... \quad (10)$$

Following equations must be satisfied by the obtained ROM:

$$t_0' = t_0 \rightarrow p_{r+1-i}' = \sum_{j=1}^{i} t_j q_{i-j}', \qquad i \in \{1,2,...\delta\}$$

$$M_i' = M_i \rightarrow p_i' = \sum M_j q_{i-j}', \qquad i \in \{1,2,...\gamma\} \quad (11)$$

Where, $\delta + \gamma = 2r$

B. Methods to Obtain Reduced Order Numerator

B1. Factor Division Algorithm

This method was first discussed by Lucas in 1983[10]. This method has an advantage that it is computationally simple. This method preserves the initial time moments and that is done without equating Padé equations. Following is a step by step procedure to acquire reduced order approximants of numerator polynomial [10]. On rewriting (1) in the following form:

$$G_n(s) = \frac{N(s)}{D(s)} = \frac{b_0 + b_1 s + b_2 s^2 + ... + b_{n-1} s^{n-1}}{(s+\lambda_1)(s+\lambda_2)....(s+\lambda_n)} \quad (12)$$

Where, $-\lambda_1 < -\lambda_2 < ...... < -\lambda_n$ represents the poles of higher order system.

Step 1: By applying factor division algorithm, reduced order numerator can be obtained as:

$$N_r(s) = \frac{N(s)}{D(s)} \times D_r(s) \quad (13)$$

Or the above equation can be rewritten as:

$$N_r(s) = \frac{N(s)}{D(s)/D_r(s)} \quad (14)$$

Step 2: Further the above equation given in step 1, can be rewritten in series expansion about $s = 0$ upto the terms of order s^{r-1}:

$$\frac{N(s)}{D(s)/D_r(s)} = \frac{\sum_{i=0}^{n-1} b_i . s^i}{\sum_{i=0}^{n-r} e_i . s^i} \quad (15)$$

The reduced order numerator can be easily obtained by using generating algorithm [5] which utilizes Routh reference formula to obtain 3rd,5th,7th etc. :

$$
\begin{array}{lllll}
 & b_0 & b_1. & b_2 \ldots b_{r-1} \\
\alpha_0 = \dfrac{b_0}{e_0} & & & \\
 & e_0 & e_1. & e_2 \ldots. e_{r-1} \\
 & q_0 & q_1 \ldots. q_{r-2} & \\
\alpha_1 = \dfrac{q_0}{e_0} & & & \\
 & e_0 & e_1 \ldots.. e_{r-2} & \\
 & r_0 & r_1 \ldots. r_{r-3} & \\
\alpha_1 = \dfrac{q_0}{e_0} & & & \\
 & e_0 & e_1 \ldots.. e_{r-3} & \\
.. & .. & & \\
.. & .. & .. & \\
 & u_0 & u_1 & \\
\alpha_{r-2} = \dfrac{u_0}{e_0} & & & \\
 & e_0 & e_1 & \\
 & v_0 & & \\
\alpha_{r-1} = \dfrac{v_0}{e_0} & & & \\
 & e_0 & &
\end{array}
\tag{16}
$$

Where,

$$q_i = b_{i+1} - \alpha_0 e_{i+1}, \qquad i = 0,1,\ldots r-2$$

$$r_i = q_{i+1} - \alpha_1 e_{i+1}, \qquad i = 0,1,\ldots r-3$$

$$\vdots \qquad \vdots$$

$$v_0 = u_1 - \alpha_{r-2} e_1 \tag{17}$$

Thus, reduced order numerator can be obtained as:

$$N_r(s) = \sum_{i=0}^{r-1} \alpha_i s^i \tag{18}$$

3.2.2 -B2. Cauer II Form

Rewrite (1) in the following form [12]:

$$G_n(s) = \frac{A_{21} + A_{22}s + A_{23}s^2 + \ldots\ldots.. + A_{2,n}s^{n-1}}{A_{11} + A_{12}s + A_{13}s^2 + \ldots\ldots + A_{1,n}s^{n-1} + A_{1,n+1}s^n} \tag{19}$$

Or

$$G_n(s) = \frac{A_{21} + A_{22}s + A_{23}s^2 + \ldots\ldots.. + A_{2,n}s^{n-1}}{(s+\lambda_1)(s+\lambda_2)\ldots\ldots(s+\lambda_n)} \tag{20}$$

Where, $-\lambda_1 < -\lambda_2 < \ldots\ldots < -\lambda_n$ are poles of higher order system.

Let the reduced order system is represented as [12]:

$$G_r(s) = \frac{B_{21} + B_{22}s + B_{23}s^2 + \ldots\ldots.. + B_{2,n}s^{r-1}}{(s+\lambda_1')(s+\lambda_2')\ldots\ldots(s+\lambda_r')} \tag{21}$$

Step 1: The reduced order approximants of denominator polynomial for elementary HOS, is acquired by using modified Padé as described in sub-section 3.1. Let the denominator polynomial is known in the following form:

$$D_r(s) = B_{11} + B_{12}s + B_{13}s^2 + \ldots. + B_{1,r+1}s^r \tag{22}$$

Step 2: To evaluate the coefficients of Cauer II form, Routh array has to be formed as below [12]:

$$
\begin{array}{lllll}
 & A_{11} & A_{12}. & \ldots. A_{1,n} & A_{1,n+1} \\
h_1 = \dfrac{A_{11}}{A_{21}} & & & & \\
 & A_{21} & A_{22} & A_{23}\ldots\ldots A_{2,n} & \\
h_2 = \dfrac{A_{21}}{A_{31}} & & & & \\
 & A_{31} & A_{32} & A_{33}\ldots\ldots\ldots\ldots\ldots & \\
h_3 = \dfrac{A_{21}}{A_{31}} & & & & \\
 & A_{41} & A_{42} & \ldots\ldots\ldots\ldots\ldots & \\
 & \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots & & & \\
.. & & & & \\
.. & \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots & & & \\
.. & & & &
\end{array}
\tag{23}
$$

Where, the first and second rows are duplicated from the numerator and denominator of the primary HOS as defined in the (1) i.e. $G_n(s)$. Remaining coefficients are evaluated by using Routh's algorithm as below [12]:

and
$$\left.\begin{aligned} A_{i,j} &= A_{i-2,j+1} - h_{i-2}A_{i-2,j+1} \\ i &= 3,4,\ldots\ldots. \\ j &= 1,2,\ldots\ldots \end{aligned}\right\} \quad (24)$$

$$h_i = \frac{A_{i,1}}{A_{i+1,1}}; \qquad i = 1,2,3,\ldots.,r \quad (25)$$

Step 3: Now on matching the coefficients of $B_{i,j}$ where, $i=1$ and $j=1,2,\ldots,r+1$ from Step 1. Coefficients of Cauer second form h_k $(k=1,2,3\ldots.,r)$ of step 2 in order to evaluate the reduced order approximants of numerator. To pursue this inverse Routh array is required to be constructed [12]:

$$B_{i+1,1} = \frac{B_{i,1}}{h_i}; \quad i=1,2,\ldots.,r \text{ and } r \le n \quad (26)$$

$$B_{i+1,j+1} = \frac{B_{i,j+1} - B_{i+2,j}}{h_i}; \qquad \left.\begin{aligned} i &= 1,2,\ldots.,(r-j) \\ j &= 1,2,\ldots.,(r-1) \end{aligned}\right\} \quad (27)$$

3.2.3 -B3. Cauer III Form

Consider the higher order system as given in (19) and reduced order form as described in (20).

Step 1: Denominator is already obtained by using modified Padé technique as described in 3.1 and the reduced order denominator is represented as in (22) [15-16].

Step 2: To perfume order reduction, first the quotients h_i and H_i of Cauer third form are to be evaluated by creating Routh array as follow [11,16]:

$$\begin{array}{lllll}
 & A_{11} & A_{12}. & \ldots.A_{1,n} & A_{1,n+1} \\
h_1 = \dfrac{A_{11}}{A_{21}} & & & & H_1 \\
 & A_{21} & A_{22} & A_{23}\ldots\ldots A_{2,n} & \\
h_2 = \dfrac{A_{21}}{A_{31}} & & & & H_2 \\
 & A_{31} & A_{32}\ldots\ldots\ldots\ldots\ldots.A_{3,n-1} & & \\
 & A_{31} & A_{32}\ldots\ldots\ldots\ldots\ldots.A_{3,n-1} & & \\
h_3 = \dfrac{A_{21}}{A_{31}} & & & & H_3 \\
 & A_{41}\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots.A_{4,n-2} & & & \\
.\,. & & & & \\
.\,. & \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots. & & & \\
.\,. & & & &
\end{array}$$

Similar to the previous method, the two rows of the above array are duplicated from the denominator and numerator respectively from the primary HOS i.e. $G_n(s)$ and remaining elements are evaluated using Routh's algorithm as follows [11]:

$$\left.\begin{aligned} A_{j,k} &= A_{j-2,k+1} - h_{j-2}A_{j-1,k+1} - H_{j-2}A_{j-1,k} \\ j &= 3,4,\ldots\ldots.,(n+1) \\ k &= 1,2,\ldots\ldots \end{aligned}\right\} \quad (29)$$

$$\text{Also, } h_i = \frac{A_{i,1}}{A_{i+1,1}} \text{ and } H_i = \frac{A_{1,n+2-i}}{A_{i+1,n+1-i}} \quad (30)$$

Where,

$i = 1,2,3,\ldots,n$

$$n = \left(\frac{r}{2}\right) + 1 \qquad \text{when r is even}$$

$$= \left(\frac{r+1}{2}\right) + 1 \qquad \text{when r is odd}$$

Step 3: Now on matching the coefficients of $B_{i,j}$ where, $i = 1,2,\ldots r$ and $j = 1,2,\ldots,r+1$ from Step 1. Coefficients of Cauer third form h_i and H_i of step 2 in order to evaluate the reduced order approximants of numerator are thus obtained by creating inverse Routh's algorithm [11] :

$$\left.\begin{aligned} B_{i+1,1} &= \frac{B_{i,1}}{h_i}; \quad i=1,2,\ldots.,r \\ B_{j+1,r+1-j} &= \frac{B_{j,r+2-j}}{H_j}; \quad j=1,2,\ldots.,(r-2); \qquad r \le n \\ B_{j-1,k+1} &= \frac{B_{j-2,k+1} - H_{j-2}B_{j-1,k} - B_{j,k}}{h_{j-2}} \\ k &= 1,2,3\ldots.,(r-2) \\ j &= 3,4,5,\ldots\ldots.,(r+1-k) \end{aligned}\right\} \quad (31)$$

IV. NUMERICAL EXAMPLES

Example 1: Consider the higher order model which is to be reduced [15-16]:

$$G(s)=\frac{2s^5+3s^4+16s^3+20s^2+8s+1}{2s^6+33.6s^5+155.94s^4+209.46s^3+102.42s^2+18.3s+1} \quad (32)$$

Determination of Reduced Order Denominator

Following are the step to find reduced order approximants for denominator polynomial for (32):

Consider the ROM of the form [14]:

$$G_r(s)=\frac{p'_1 s+p'_2}{s^2+q'_1 s+q'_2} \quad (33)$$

Step 1: Expand (32) around $s=0$ and $s=\infty$

$$=1-10.3s+106.07s^2+......$$
$$=1s^{-1}-15.3s^{-2}+187.27s^{-3}+...... \quad (34)$$

Step 2: On expanding (6) around $s=0$ and $s=\infty$:

$$=t_1'+t_2's+t_3's^2+....$$
$$=M_1's^{-1}+M_2's^{-2}+M_3's^{-3}+... \quad (35)$$

Step 3: To get perfect matching of responses, it is required that 2r terms are matched:

$$t_1=t_1'$$
$$M_1=M_1'$$

$$t_2=t_2'$$
$$M_2=M_2' \quad (36)$$

Step 4: Above assumptions and following equations can be used to achieve a second order reduced model .Using following equations variables for ROM (32) can be achieved as:

$$t_1'=\frac{p_2'}{q_2'}$$
$$t_2'=\frac{p_1'-t_1'q_1'}{q_2'}$$

$$M_1'=p_1'$$
$$M_2'=p_2'-M_1'q_1' \quad (37)$$

For effective and better time response approximation, it is significant to give equal cogitation to both time moments and markov parameters. As explained earlier that 2r terms are needed to be matched, also must satisfy $\delta+\gamma=2r$, hence $\gamma=\delta=2$ is essential to be considered.

Using (33) and (37):

$$p_1'=1$$
$$p_2'=1.53$$

$$q_1'=16.83$$
$$q_2'=1.53 \quad (38)$$

Thus required reduced order denominator is obtained as:

$$D^r(s)=s^2+16.83s+1.53 \quad (39)$$

Determination of Reduced Order Numerator

Step 5: By using factor division algorithm as described in section II, following values are obtained to find the reduced order numerator:

$$b_0=1.53;\quad b_1=29.07;\quad b_2=166.24;\quad b_3=369.08;\quad e_0=1.08 \quad (40)$$

By using above parameters, the values for α_0 and α_1 are obtained as 1.53 and 1.08 respectively. Further on expending (18), the reduced order numerator is expressed as:

$$N_r(s)=\alpha_0+\alpha_1(s)$$
$$=1.53+1.08s \quad (41)$$

Hence the reduced order system for original HOS using factor division algorithm by combining (39) and (41) is expressed as follows:

$$G_r(s)=\frac{N_r^1(s)}{D_r(s)}=\frac{1.53+1.08s}{1.53+16.83+s^2} \quad (42)$$

Step 6: By using Cauer II form as described in (23)-(27), following values are obtained to find the reduced order numerator:

$$h_1=1;\quad h_2=0.09 \quad (43)$$

$$B_{11}=1.53;\quad B_{12}=16.83;$$
$$B_{13}=1;\quad B_{31}=17$$
$$B_{21}=1.53;\quad B_{22}=-0.17 \quad (44)$$

By using above parameters value the reduced order numerator is expressed as:

$$N_r(s)=B_{21}+B_{22}(s)$$
$$=1.53-0.17s \quad (45)$$

Hence the reduced order system for original HOS using Cauer II form by combining (39) and (45) is expressed as follows:

$$G_r(s)=\frac{N_r^2(s)}{D_r(s)}=\frac{1.53-0.17s}{1.53+16.83+s^2} \quad (46)$$

Step 7: By using Cauer III form as described in (28)-(31), following values are obtained to find the reduced order numerator:

$$h_1=1;\quad h_2=0.107;\quad H_1=1 \quad (47)$$

$$B_{11} = 1.53; \quad B_{12} = 16.83;$$
$$B_{13} = 1; \quad B_{31} = 14.23$$
$$B_{21} = 1.53; \quad B_{22} = 1.07 \tag{48}$$

By using above parameters value the reduced order numerator is expressed as:

$$\begin{aligned} N_r(s) &= B_{21} + B_{22}(s) \\ &= 1.53 + 1.07s \end{aligned} \tag{49}$$

Hence the reduced order system for original HOS using Cauer III form by combining (39) and (49) is expressed as follows:

$$G_r(s) = \frac{N_r^3(s)}{D_r(s)} = \frac{1.53 + 1.07s}{1.53 + 16.83 + s^2} \tag{50}$$

Example 2: Consider the higher order model which is to be reduced [7]:

$$G(s) = \frac{s^3 + 7s^2 + 24s + 24}{s^4 + 10s^3 + 35s^2 + 50s + 24} \tag{51}$$

Determination of Reduced Order Denominator

Step: 1 On using (4)-(11), the reduced order denominator for example 2 is obtained as:

$$D^r(s) = s^2 + 27.096s + 24.096 \tag{52}$$

Determination of reduced order numerator

Step 2: By using factor division algorithm as in (12)-(18), reduced order numerator is obtained as follows:

$$\begin{aligned} N_r(s) &= \alpha_0 + \alpha_1(s) \\ &= 24.096 + 1.s \end{aligned} \tag{53}$$

Hence the reduced order system for original HOS using factor division algorithm by combining (52) and (53) is expressed as follows:

$$G_r(s) = \frac{N_r^1(s)}{D_r(s)} = \frac{24.096 + 1.s}{24.096 + 27.096.s + s^2} \tag{54}$$

Step 3: As described in Cauer II algorithm from (19)-(27), reduced order numerator is obtained as:

$$\begin{aligned} N_r(s) &= B_{21} + B_{22}.s \\ &= 24.096 + 0.99s \end{aligned} \tag{55}$$

Thus the transfer function is obtained by combining (52) and (55) as:

$$G_r(s) = \frac{N_r^2(s)}{D_r(s)} = \frac{24.096 + 0.99.s}{24.096 + 27.096.s + s^2} \tag{56}$$

Step 4: Deploying Cauer III form algorithm (28)-(31), reduced order numerator is thus acquired as:

$$\begin{aligned} N_r(s) &= B_{21} + B_{22}(s) \\ &= 24.096 + 0.99s \end{aligned} \tag{57}$$

Consequently, reduced order model is obtained by combining (52) and (57) as:

$$G_r(s) = \frac{N_r^2(s)}{D_r(s)} = \frac{24.096 + 0.992.s}{24.096 + 27.096.s + s^2} \tag{58}$$

V. SIMULATION RESULTS

This section renders the simulation results for the examples under consideration. The simulation results obtained for both the examples are further compared with existing techniques to evince the effectiveness and powerfulness of the suggested methods. Responses for each example are compared with existing methods, on the basis of an error index which is known as integral square error (ISE).

The step responses of the original higher order system and reduced order system in (42), (46) and (50), for Example 1, is illustrated in Fig. 1-3 respectively. Along with the response curves obtained using proposed hybrid methods and original method, the responses obtained from various existing methods are also shown in the same figure, which makes it easy to draw a better comparison among diminution methods. These responses clearly show that the proposed method provides sufficiently approximated result for original higher order system.

Moreover, to validate the excellent approximation and better efficiency of the suggested model, a comparison on the basis of integral square error with recently proposed methods, is also made available in the Table I. The existing methods are compared with well known conventional methods and also fusion of both conventional and new trends in the field of MOR i.e. metaheuristic based approaches. From Fig .1-3 and Table I, it can be deduced that the proposed hybrid methods provide most approximated result than the existing methods.In a similar fashion simulation for Example 2 is performed. In Figure 4-6, responses of reduced order system and original HOS, responses of the reduced order model obtained using existing techniques are also illustrated in the same figures. Table II represents a comparison among various methods and proposed methods to validate the uniqueness and their performance on the basis of performance index known as ISE. The above mentioned figure clearly depicts that the proposed diminution techniques perform well and yields contiguous response to that of the original HOS.

VI. STABILITY OF THE REDUCED ORDER MODEL

The suggested methods in this paper persuade the obtained lower order system in each case to stable model if the original system is stable. Due to this intrinsic attribute of the suggested methods, the acquired low order models are stable as evinced in Example 1and 2.

In each suggested method the reduced order approximants are obtained using Padé based moment matching i.e. matching of both time moments and Markov parameters. Padé based moment matching is benefited by dominant pole retention. Hence it preserves the stability and all the poles of lower order system lie in the left half of the s-plane. Thus produced system will always be stable provided that the original system is stable.

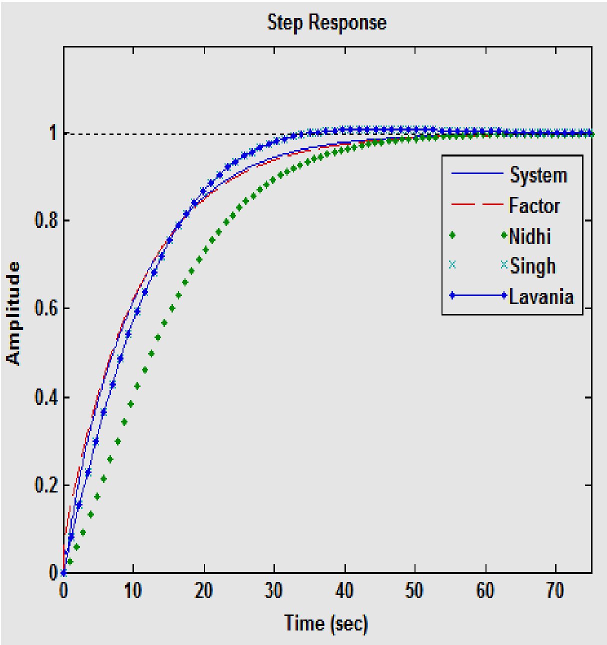

Fig. 1. Comparison of step responses of HOS and suggested (Factor Division Algorithm) technique with existing methods.

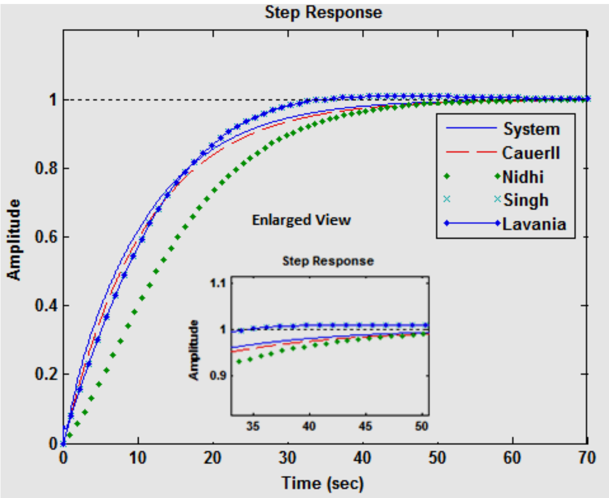

Fig. 2. Comparison of step responses of HOS and suggested (Cauer II Form) method with existing methods

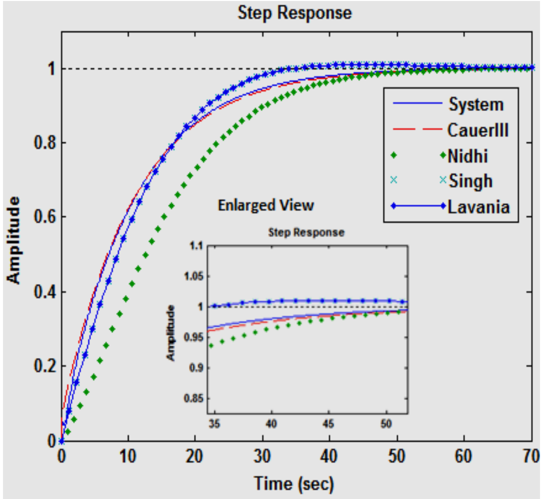

Fig. 3. Comparison of step responses of HOS and suggested (Cauer III Form) method with existing methods

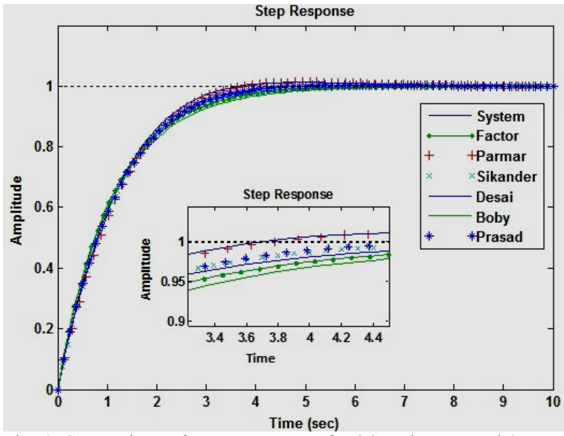

Fig. 4. Comparison of step responses of HOS and suggested (Factor Division Algorithm) technique with existing methods

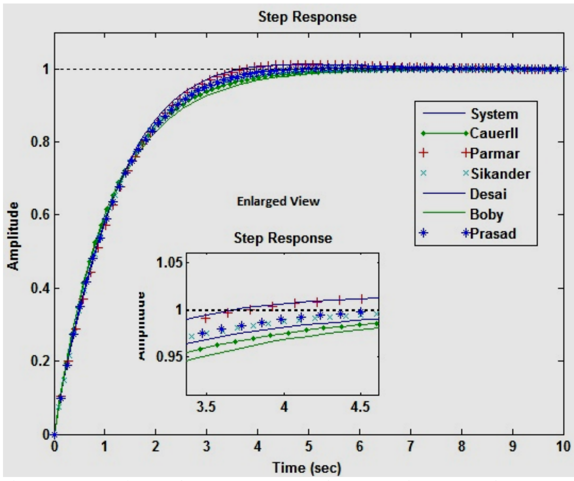

Fig. 5. Comparison of step responses of HOS and suggested (Cauer II Form) technique with existing methods

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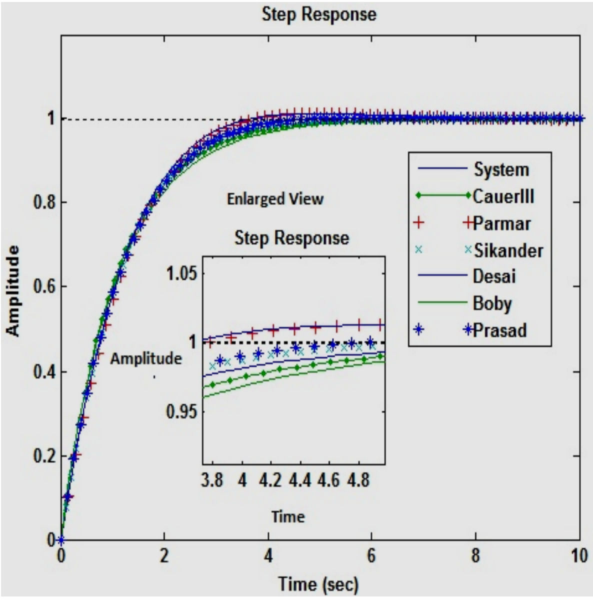

Fig. 6. Comparison of step responses of HOS and suggested (Cauer III Form) technique with existing methods

TABLE I: COMPARISON ON THE BASIS OF PERFORMANCE INDEX ISE FOR EXAMPLE 1

METHOD	MODEL	ISE
Original	$G(s)=\frac{2s^5+3s^4+16s^3+20s^2+8s+1}{2s^6+33.6s^5+155.94s^4+20 +102.42s^2+18.3s+1}$	--
Proposed Method 1	$G_2(s)=\frac{1.08s+1.53}{s^2+16.83s+1.53}$	9.7828 e-05
Proposed Method 2	$G_2(s)=\frac{-0.17s+1.53}{s^2+16.83s+1.53}$	2.2528 e-04
Proposed Method 3	$G_2(s)=\frac{1.07s+1.53}{s^2+16.83s+1.53}$	9.5523 e-05
Singh (2008)[15]	$G_2(s)=\frac{5.9979s+1}{87.97s^2+15.96s+1}$	0.0650
Mahmoud et.al (1981)[16]	$G_2(s)=\frac{0.0227s+0.0131}{s^2+0.22s+0.0131}$	8.7854
Lavania et.al.(2015)[21]	$G_2(s)=\frac{0.667s+1.503}{s^2+16.83s+1.53}$	4.166 e-004

TABLE II: COMPARISON ON THE BASIS OF PERFORMANCE INDEX ISE FOR EXAMPLE 2

METHOD	MODEL	ISE
Original	$G(s)=\frac{s^3+7s^2+24s+24}{s^4+10s^3+35s^2+50s+24}$	--
Proposed Method 1	$G_2(s)=\frac{1s+24.096}{s^2+27.096s+24.096}$	**1.8748 e-04**
Proposed Method 2	$G_2(s)=\frac{0.99s+24.096}{s^2+27.096s+24.096}$	**1.855 e-04**
Proposed Method 3	$G_2(s)=\frac{0.992s+24.096}{s^2+27.096s+24.096}$	**1.8593 e-04**
Desai and Prasad (2013a)[17]	$G_2(s)=\frac{0.8058s+0.7949}{s^2+1.65.s+0.7944}$	**2.18 e-04**
Sikander (2015)[2]	$G_2(s)=\frac{0.7751.s+1.258}{s^2+2.12s+1.258}$	**5.77 e-05**
Parmar (2007)[18]	$G_2(s)=\frac{0.74425755s+0.699157}{s^2+1.45771s+0.6997}$	**1.65 e-03**
Boby and Pal (2010)[19]	$G_2(s)=\frac{0.9315s+1.092}{s^2+2.756s+1.6092}$	**2.78 e-03**
Desai and Prasad (2013b)[20]	$G_2(s)=\frac{0.8s+0.686}{s^2+1.47s+0.686}$	**3.5 e-04**

VII. CONCLUSIONS

A collection of methods for the diminution of large scale systems have been proposed. These methods utilizes Factor division algorithm, Cauer II form and Cauer III form for evaluation of low order numerator approximants for HOS, whereas modified Padé i.e. matching of both Markov parameter and time moments for determination of reduced order approximants for denominator.

In brief, this paper describes three methods for diminution of large scale systems as follows:

- Factor division algorithm and modified Padé technique
- Cauer II form and modified Padé technique
- Cauer III form and modified Padé technique

Results acquired from suggested methods are represented in the form of time response curves in Figure 1-6 to evince the efficacy of the suggested methods. Also to advance the comparison among methods, a performance index ISE is chosen. In Table I and II, the suggested methods and well known existing methods are thus compared. The lower values of ISE obtained for each suggested method manifest the worthiness and powerfulness of the methods. The all suggested methods preserve stability for reduced order system provided that original HOS is stable. To validate the effectiveness and authenticity of suggested methods, two different numerical examples are solved for each proposed technique. Results obtained from proposed methods validate them as influential and powerful for reduction of large scale system. Further, all suggested methods can be extended for MIMO systems.

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