Probabilistic Load Flow Analysis of Power System Network Considering Uncertainty with Generation and Correlated Loads

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***Abstract* - Probabilistic load flow has gain attractive attention in electric power system planning, operation, and control. Probabilistic load flow is an efficient tool to access the performance of the power system considering the uncertainties of the load demand and generation. However, future operating condition and operating criteria could not be predicted without considering uncertainty. Due to integration of renewable energy sources into power system network brought uncertainty and dependence factor. Most of the uncertainties in power system are load increment, generation scheduling, and network topology in which engineers are dealing with them. In this paper, Box-Muller sampling algorithm is proposed to solve the probabilistic load flow problems considering uncertainty with generation and correlated loads. The main advantage of the proposed approach is that accurate solution can be obtain with less computation time and can address the dependence between variables. Also, it is almost unconstrained for the probability distributions of the input random variables. The proposed method is compared with correlated simple random sampling Monte Carlo simulation. For the demonstration purpose, the proposed method is investigated using IEEE 14-bus and IEEE 118-bus test systems. The simulation results indicate that Box-Muller sampling method is a promising approach in probabilistic load flow evaluation.**

Keywords - Box-Muller sampling; Monte Carlo simulation; Probabilistic load flow; Power system; Random variable.

I. INTRODUCTION

In an electric power distribution system, electric power flow is from the substation to the load points. Distribution network has a number of buses and branches. The flow of active and reactive power flow through this network is called the load flow. In load flow study, bus node voltage, currents through each branch, branch losses, active and reactive power flow through different branches are the parameters to be known. Load flow calculation is fundamental analysis for the operation and planning engineers to determining the unknown parameters. Distribution network provides power to a variety of loads, such that residential, industrial, and commercial, etc. Most of the loads are a specific range of variations over the time. The load profile of the feeder is cyclic phenomena. Actually, consumer loads are not a planned loads, but it will change in a cyclic pattern. Most of the engineering problems in distribution network are subject to deal with uncertainties especially due to renewable sources. The infeed uncertainties with smart grid can be a model as a randomness in modelling approach [1].

Uncertainties may be daily load variation, generation outage, faults, and failures in power system network. To see the better output of these uncertainties, a probabilistic approach is an excellent way. For the effective designing and operation of power system uncertainties modelling tools are very important [2]. In deterministic load flow (DLF) study, certain load and generation are used to investigate the state variable of the network. To handle these uncertainties probabilistic load flow (PLF) study are taken into consideration [3]. In PLF, future condition of the power system is investigated by using different input parameters. In PLF studies, power system parameters are taken as random variables i.e. Generations outages and loads. Hence, the possible output results are identified by changing the parameters.

Broadly speaking, there are three main categories of methods in PLF evaluation: analytical methods, approximate methods, and Monte Carlo simulation (MCS) methods [4-7]. Firstly analytical approach for PLF study, in this approach power flow equations are linearized to make working with probability function. In [8-10] analytical approaches are used to determine the probabilistic load flow analysis. Secondly, Approximate approaches [10, 11], did not require the linearization of load flow equations. In these approaches, a number of evaluation points have been decreased to minimise the computation burden and storage. The third approach is by using the MCS methods. In MCS, there is no need to an approximation of load flow problems. MCS approaches are based upon the repetition process. A number of research articles have been published by using the MCS in the probabilistic study of the power system [8, 10, 12, 13]. The drawback of MCS is a large number of iteration, time, and ample storage. These drawbacks are overcome by the development of fast and efficient computer now-a-day. That is why MCS method is widely used in the PLF studies still [14, 15].

MCS handled uncertain problems through a series of deterministic calculation and recognised as a robust and most accurate method. The Simple Random Sampling (SRS) with MCS has been used widely for benchmark for other methods to determine the accuracy and robustness, but this suffers heavy computation burden and ample storage [4]. All of the PLF analysis approaches, the primary task is to determine the computational efficiency and results accuracy. So that the prediction about the power system network should be precisely and accurately for planning and operation purpose. In this work, two factor are adopted for creating uncertainty in proposed network like generation and dependent loads. Similarly, two factor is calculated as output parameters like computational efficiency and accuracy. Box-Muller sampling method with MCS is adopted to establish the probability distribution of statistical dependencies with input random variables. The loads are modelled as a Gaussian distribution with correlated input random variables and generation outage is modelled as binomial distribution. The results show that the proposed method can give an accurate solution with less computation burden, more robust, and almost unconstraint for probability distributions.

The rest of the paper is organized as follows: Section 2 provides an introduction to Monte Carlo simulation method and Box-Muller algorithm with its detailed description. Performance analysis is discussed in section 3. Finally, section 4 provides the conclusion of the paper.

II. PROPOSED METHODS

A. *Monte Carlo Simulation Basic Concept*

MCS is a technique used to understand the impact of output variables in case of uncertainty with input variables in a forecasting model. When we develop a forecasting model that plans to ahead for future, we make a certain assumption. This assumption may be a return on a portfolio, the cost of construction, how long to complete the task, etc. Actually, these tasks will be held in future. The expected value of the certain parameters are to be calculated.

In MCS, the results are available in probability distribution form. The wide range of output results, make it easy to understand the risk and uncertainty in the proposed model. The fundamental steps of MCS are: a random number is selected based upon the range of estimates. Based on this random number model is calculated. The results are recorded and repeat the process. This repetition is hundreds or thousands of time, each time using the different randomly selected value of input variable. When the simulation is completed, a number of output results from the model calculation are obtain. Each output results is based upon the random value of input variable. These results are used to describe the likelihood of output variable based upon the input variable.

B. *Box-Muller algorithm and its detailed description*

In this section, step-by-step implementation of probabilistic load flow analysis is discussed.

Step 1: Firstly, we prepare a CDF for all input random variables X_i (i =1, ---------, n) and correlation matrix C_X .The loads are considered as a normally distributed random variable. These loads have a mean that is base value and standard deviation. We can change the value of standard deviation. In the proposed model 5% standard deviation has been used for calculating the results. The Pearson's linear correlation coefficient is used to calculating the correlation coefficient between loads.

Let C_X be the correlation matrix of the n input random variables as X_1, X_2, ……, X_n, as shown in (1):

$$C_X = \begin{bmatrix} 1 & \rho_{12} & \cdots & \rho_{1n} \\ \rho_{21} & 1 & \cdots & \rho_{2n} \\ \cdots & \cdots & \cdots & \cdots \\ \rho_{n1} & \rho_{n2} & \cdots & 1 \end{bmatrix} . \quad (1)$$

$$\rho_{12} = \frac{con\left(X_i, X_j\right)}{\sigma_i \sigma_j} . \quad (2)$$

where σ_i and σ_j are the standard deviation of X_i and X_j, con (X_i, X_j) is the covariance of X_i and X_j , and ρ_{ij} is the correlation coefficient of X_i and X_j. Each element of C_x should be calculated by (2) and non-diagonal elements of C_x must be follow (3):

$$\rho_{ij} = \iint_{-\infty}^{+\infty} \left(\frac{x_i - x_i}{\sigma_i}\right)\left(\frac{x_j - x_j}{\sigma_j}\right) \times f\ x_i x_j \left(x_i, x_j\right) dx_i dx_j . \quad (3)$$

Step 2: For convergence of MCS, we need to select the number of iteration and sampling to the evolution of fitness function. A maximum number of iteration and number of sampling are set in this step.

Step3: In this step, we determined the sampling for a specific iteration. In order to determine the sampling, we need a random number that should be generated. In this model Box-Muller algorithm equation is used to generate the sampling for each iteration by using (4).

$$S_i = \mu_i + \cos\left(2\pi u_1\right) \times \sigma_i \sqrt{-2\ln\left(u_2\right)}. \quad (4)$$

where,

S_i= i_{th} sample

μ= mean value of random variable

σ= standard deviation of a random variable

u_1 & u_2 = uniformly distributed random number in range (0, 1)

For the proposed PLF model, μ_i and σ_i are the base load and standard deviation for input variable.

Step 4: In this step, the fitness function is calculated by using determined sample. In PLF analysis, the fitness function is a deterministic load flow analysis.

Step 5: In this step, sampling criteria are verified by a maximum number of samples. If a maximum number of samples is not reached then PLF algorithm fellow the step.3 otherwise it continues.

Step 6: In this step, expected value of output variable are calculated related to each iteration. In our case, bus voltage magnitude and transmission line loading is an output variable.

Step 7: In this step, termination criteria are verified. Termination criteria are the maximum number of iterations that is selected as an input parameter that is selected at starting time of the algorithm. The algorithm will terminate when it reached at maximum iteration otherwise it follow the step 3.

III. Performance Analysis of the Box-Muller Method

The flowchart of the proposed method is shown in Fig. 1. To investigate the performance of the Box-Muller method, an IEEE 14-bus and an IEEE 118-bus [16] test system are used in the "DIgSILENT PowerFactory platform. The simulation was performed on PC with AMD A12-9700P, RADEON R7, 10 COPMPUTE CORES UC+6G, 2.5 GHZ processing speed, and 8 GB RAM". The results obtained by the proposed method are compared with correlated simple random sampling (CSMCS).

The results obtained by the CSMCS method with sample size of 20,000 are assumed to be accurate and consider as a benchmark. The mean and standard deviation of CSMCS method are express as μ_{acc} and σ_{acc} respectively. Similarly, the mean and standard deviation of proposed method are express as μ_{pro} and σ_{pro} respectively. The error indices [4, 10, 17], of the output random variables are calculated as follow:

$$\varepsilon_{\mu}^{*} = \left|\frac{\mu_{acc} - \mu_{pro}}{\mu_{acc}}\right| \times 100\% . \quad (5)$$

$$\varepsilon_{\sigma}^{*} = \left|\frac{\sigma_{acc} - \sigma_{pro}}{\sigma_{acc}}\right| \times 100\% . \quad (6)$$

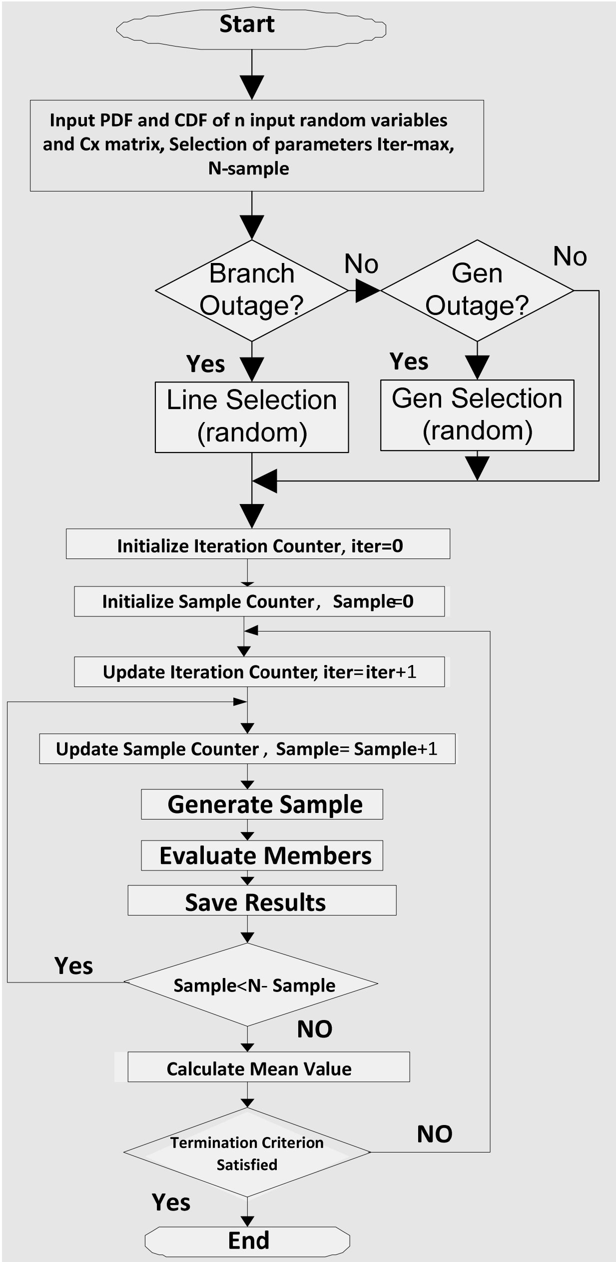

Figure 1. Basic Flow chart of MCS with Box-Muller method.

The output random variables are divided into four categories: node voltage magnitude *V*, node voltage phase angle *θ*, line active power *P*, and line reactive power *Q*. Two error indices are introduced, ε_{μ}^{*} is error of the mean value of the output random variables with * category (* represents *V, θ, P,* and *Q*). Second is ε_{σ}^{*} error of standard deviation with * category.

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A. IEEE 14-Bus Test System.

IEEE 14-bus test system are shown in Fig. 2 and all of its deterministic data is available in [16]. Load parameters are shown in Table I. In this test system total 26 input random variables. The constant voltage model is adopted for simulation purpose. The active power and reactive power of the loads are following the normal distribution. The loads in the system are divided into three group according to their characteristics. First group is from 2 to 6, and second group is from 10 to 14, and load number 9 is consider as alone. Loads in same group is dependent but independent to other groups. The correlation between first and second group are presented in (7) and (8), respectively.

In the simulation, proposed model is calculated 1000 time with step size 100. The variation of the four category output random variable are calculated. The calculated error indices like mean and standard deviation of output variable determine the convergence degree of entire system. The error curves about mean and standard deviation of voltage at certain bus are shown in Fig. 3(a) and (b) respectively. The error curves about mean and standard deviation of voltage angle are shown in Fig. 3(c) and 3(d) respectively. Similarly, the error curves of active power and reactive power through the line 2-3 are shown in Fig. 4. All of the error indices are calculated only for specific bus and line, the remaining error indices for other buses and lines are similar. The average error index with 100 times simulation express as AVG_{100} and average error index with 1000 sample size expressed as AVG_{1000} are calculated for each output variable for both methods, presented in Table II. Similarly, the average standard deviation error index with 100 times simulation express as $StdDev_{100}$ and average standard deviation error index with 1000 sample size express as $StdDev_{1000}$ for each output variable are calculated for both methods, presented in Table II. The results of Table II show that all the error index are within 5% for proposed method but for benchmark system the error indices are more than 5%. This show that the proposed method is more convergent and stable method.

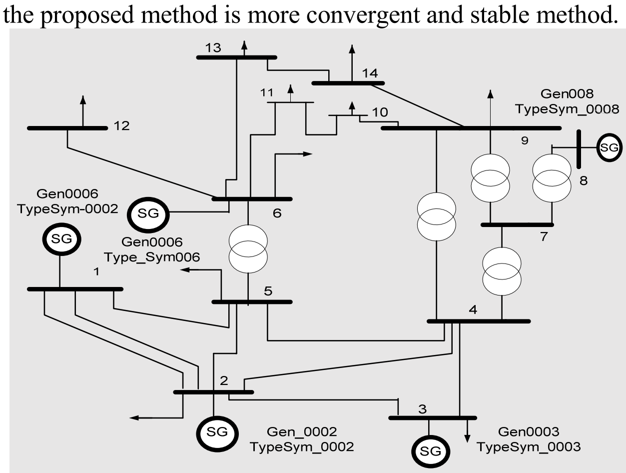

Figure 2. IEEE 14-bus test system

The probability density function (PDF) and cumulative distribution function (CDF) of the active power through the line 2-3 are shown in Fig. 5 for both methods. The PDF and CDF of reactive power through the line 2-3 are shown in Fig. 6 for both methods. From these figures, it is clear that the proposed method is same accurate but with less sample size. The computational time for both methods are compare in Fig. 8(a) for proposed test system with 100 step size. Finally, the proposed method is accurate as well as efficient computational property.

TABLE I. PARAMETERS OF LOADS

Node No.	Mean (p.u)	StdDev (p.u)	Power Factor
2	0.215	0.065	0.864
3	0.943	0.031	0.991
4	0.481	0.115	0.365
5	0.074	0.036	0.978
6	0.111	0.037	0.833
9	0.295	0.156	0.872
10	0.094	0.036	0.823
11	0.036	0.015	0.889
12	0.061	0.024	0.965
13	0.134	0.048	0.919
14	0.151	0.062	0.949

$$C_{Group-1} = \begin{bmatrix} 1 & 0.6 & 0.7 & 0.5 & 0.7 \\ 0.6 & 1 & 0.7 & 0.6 & 0.8 \\ 0.7 & 0.7 & 1 & 0.7 & 0.8 \\ 0.5 & 0.6 & 0.7 & 1 & 0.7 \\ 0.7 & 0.8 & 0.8 & 0.7 & 1 \end{bmatrix}. \qquad (7)$$

$$C_{Group-2} = \begin{bmatrix} 1 & 0.5 & 0.6 & 0.7 & 0.6 \\ 0.5 & 1 & 0.6 & 0.9 & 0.8 \\ 0.6 & 0.6 & 1 & 0.6 & 0.8 \\ 0.7 & 0.9 & 0.6 & 1 & 0.8 \\ 0.6 & 0.8 & 0.8 & 0.8 & 1 \end{bmatrix}. \qquad (8)$$

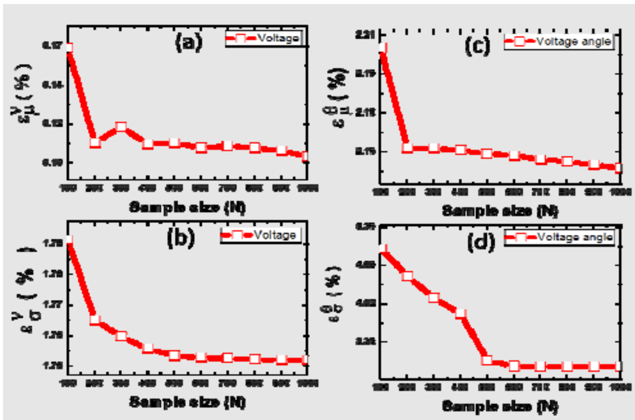

Figure 3. Error curves of the voltage mean (a), voltage standard deviation (b), voltage angle mean (c), and voltage angle standard deviation (d) at bus 7, IEEE 14-bus test system

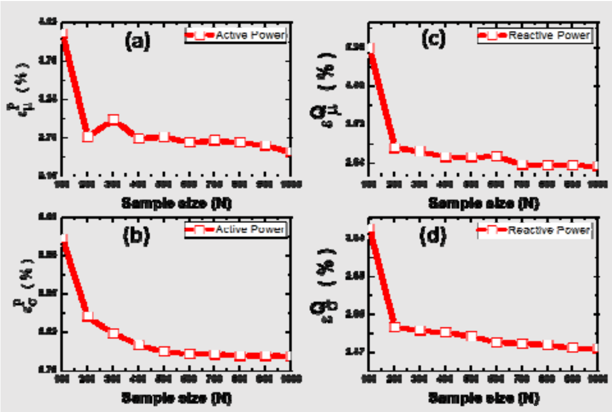

Figure 4. Error curves of the active power mean (a), active power standard deviation (b), reactive power mean (c), and reactive power standard deviation (d) through line 2-3, IEEE 14-bus system

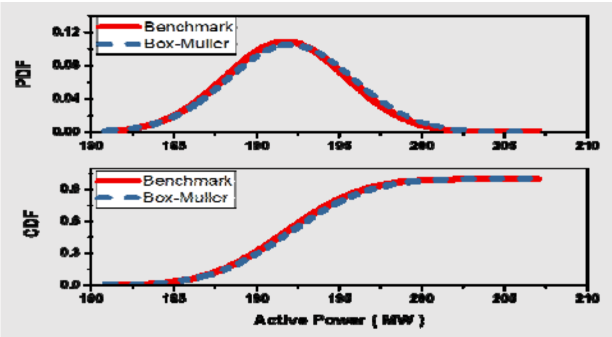

Figure 5. PDF and CDF comparison of active power through the line 2-3 (MW), IEEE 14-bus system.

TABLE II. THE EXPECTED ERROR COMPARISON OF TWO METHODS FOR IEEE 14-BUS TEST SYSTEM (N=1000)

Method		SMCS	Box-Muller
ε_{μ}^{V} (%)	AVG_{1000}	0.189	0.099
	AVG_{100}	0.278	0.166
ε_{σ}^{V} (%)	$StdDev_{1000}$	1.755	1.759
	$StdDev_{100}$	1.776	1.781
$\varepsilon_{\mu}^{\theta}$ (%)	AVG_{1000}	2.145	2.151
	AVG_{100}	2.198	2.204
$\varepsilon_{\sigma}^{\theta}$ (%)	$StdDev_{1000}$	2.158	2.159
	$StdDev_{100}$	4.644	3.015
ε_{μ}^{Pij} (%)	AVG_{1000}	4.865	2.652
	AVG_{100}	6.095	4.281
$\varepsilon_{\sigma}^{Pij}$ (%)	$StdDev_{1000}$	2.384	2.395
	$StdDev_{100}$	4.875	3.889
ε_{μ}^{Qij} (%)	AVG_{1000}	2.565	2.561
	AVG_{100}	7.01	3.26
$\varepsilon_{\sigma}^{Qij}$ (%)	$StdDev_{1000}$	2.398	2.406
	$StdDev_{100}$	3.091	3.087

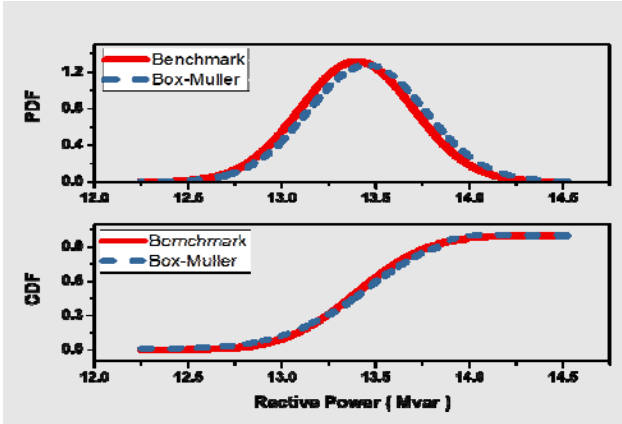

Figure 6. PDF and CDF comparison of reactive power through the line 2-3 (Mvar), IEEE 14-bus system

B. IEEE 118-Bus Test System.

All of the deterministic data of IEEE 118-bus test system are introduced in [16]. The probabilistic assumption about the data is same as for IEEE 14-bus test system. There are total 170 input random variables. Loads are assumed to be follow the normal distribution and generator output is binomial distribution. Same like a first case CSMCS is consider as accurate with 20,000 simulations.

TABLE III. THE EXPECTED ERROR COMPARISON OF TWO METHODS FOR IEEE 118-BUS TEST SYSTEM (N=1000)

Method		CSMCS	Box-Muller
ε_{μ}^{V} (%)	AVG_{1000}	0.083	0.086
	AVG_{100}	0.144	0.156
ε_{σ}^{V} (%)	$StdDev_{1000}$	1.595	1.654
	$StdDev_{100}$	1.598	1.671
$\varepsilon_{\mu}^{\theta}$ (%)	AVG_{1000}	2.106	2.114
	AVG_{100}	2.308	2.335
$\varepsilon_{\sigma}^{\theta}$ (%)	$StdDev_{1000}$	2.143	2.162
	$StdDev_{100}$	2.137	2.149
ε_{μ}^{Pij} (%)	AVG_{1000}	0.843	0.854
	AVG_{100}	4.986	5.113
$\varepsilon_{\sigma}^{Pij}$ (%)	$StdDev_{1000}$	2.341	2.441
	$StdDev_{100}$	6.744	4.756
ε_{μ}^{Qij} (%)	AVG_{1000}	2.521	2.532
	AVG_{100}	5.241	4.245
$\varepsilon_{\sigma}^{Qij}$ (%)	$StdDev_{1000}$	2.308	2.355
	$StdDev_{100}$	3.056	3.068

The error indices of four category (*V*, *θ*, *P*, *Q*) in IEEE 118-bus test system by proposed method are calculated with 1000 simulation. The mean and standard deviation error pattern of four output variables were almost similar to previous test system. It is not shown here due to concise of space. The expected error comparison of proposed test system is presented in Table II for both methods. The results of Table II shows that mean of the error with 100 samples and with 1000 samples are within 5% error but more than 5% error with CSMCS. The PDF and CDF curve of the active power through the line 78-79 are shown in Fig. 7. It is shown that curves are close as compare to previous test system. The computation time of proposed test system is compare in Fig. 8(b) for both methods for IEEE 118-bus test system. The computational time is slightly increased due to increase of input random variables.

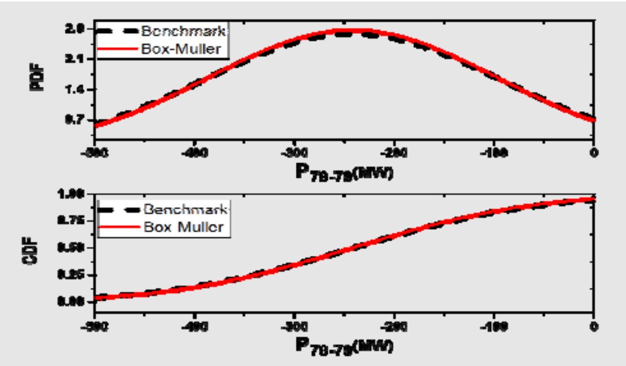

Figure 7. PDF and CDF comparison of active power through the line 75-76 (MW), IEEE 118-bus system

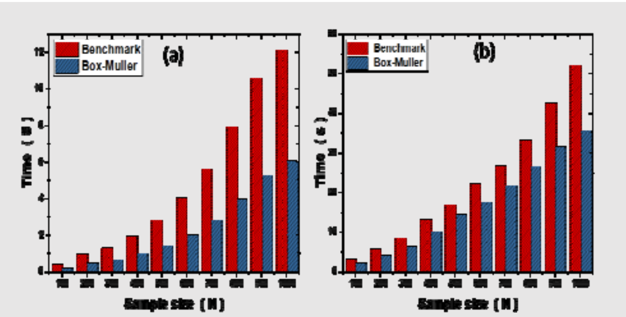

Figure 8. Comparison of computation time (a) IEEE 14-bus test system (b) IEEE 118-bus test system.

IV. Conclusion

This paper has proposed the use of the Box-Muller sampling algorithm with Monte Carlo simulation for solving the PLF problems with power system network. The two test system have illustrated that proposed method can achieve a better sampling efficiency than CSMCS and make it possible for Monte Carlo simulation to get an accurate simulation result with a much smaller simulation size. This is because proposed method is an efficient sampling method to ensure the coverage of specific distribution of input random variables. The error indices obtain by proposed method are comparable with CSMCS but the computation time is significant. The proposed method is flexible for any known distribution, can address all possible uncertainties in load, and provide more reliable distribution of output data. The robustness and flexibility of the proposed method enable it to become a sampling method with a tremendous potential to be applied to be many power system probabilistic problems. The proposed method is simple and accurate in handling correlated variables.

Acknowledgments

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