Signed Product Cordial Labeling for Some Families of Graphs

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***Abstract* - A vertex labeling of graph G is a function $f: V(G) \to \{-1,1\}$ with an induced edge labeling $f^*: E(G) \to \{-1,1\}$ defined by $f^*(uv) = f(u)f(v)$ is called a signed product cordial labeling if $|v_f(-1) - v_f(1)| \le 1$ and $|e_{f^*}(-1) - e_{f^*}(1)| \le 1$, where $v_f(-1)$ and $v_f(1)$are the number of vertices labeled with -1 and +1 respectively and $e_{f^*}(-1)$ and $e_{f^*}(1)$are the number of edges labeled with -1 and +1 respectively. A graph G is signed product cordial if it admits signed product cordial labeling. In this paper we proved the existence of signed product cordial labeling for some families of graphs.**

Keywords - Graphs, labeling, bijective function.

I. INTRODUCTION

A graph labeling is an assignment of integers to the vertices or edges or both subject to certain conditions. Labeled graphs are becoming an increasingly useful family of mathematical models from a broad range of applications. Baskar Babujee and Shobana [2] introduced the notion of signed product cordial labeling. A vertex labeling of graph G $f: V(G) \to \{-1,1\}$ with an induced edge labeling $f^*: E(G) \to \{-1,1\}$ defined by $f^*(uv) = f(u)f(v)$ is called a signed product cordial labeling if $\left|v_f(-1) - v_f(1)\right| \le 1$ and $\left|e_{f^*}(-1) - e_{f^*}(1)\right| \le 1$, where $v_f(-1)$ is the number of vertices labeled with -1, $v_f(1)$ is the number of vertices labeled with 1, $e_{f^*}(-1)$ is the number of edges labeled with -1 and $e_{f^*}(1)$ is the number of edges labeled with 1. A graph G is signed product cordial if it admits signed product cordial labeling. For a survey on graph labeling, we refer to Gallian [4].In this paper we proved the existence of signed product cordial labeling for some families of graphs.

Definition 1.1 [8] Duplication of an edge $e=v_iv_{i+1}$ by a vertex v' in a graph G produces a new graph G' such that $N(v')=\{v_i,v_{i+1}\}$.

Definition 1.2 [3] A shell S_n is the graph obtained by taking (n-3) concurrent chords in a cycle C_n on n vertices. The vertices at which all the chords are concurrent is called the apex vertex.

Definition 1.3 [6] Twig graph is a graph obtained from a path graph $P_n(n\ge3)$ by attaching exactly two pendant edges to each internal vertex of P_n. It is denoted by T_m where m is the number of internal vertices of the path graph.

Definition 1.4 [1] A Y tree is a tree obtained from the path by appending an edge to a vertex of the path adjacent to an end point.

Definition 1.5 [5] A web graph is a graph obtained from C_n and iterating the process of adding pendant points and joining them to form a cycle and then adding pendant points to the new cycle. It is denoted by w(t,n) where t denotes the number of cycles.

Editorial note: where it is necessary to change from 2-column to 1-column format to accommodate long equations the symbol <= => will be used to mark the start and end of the 1-column format, except where it is clear e.g. after wide tables.

II. MAIN RESULTS

Theorem 2.1: The duplication of all edges by vertices in a wheel graph W_n, $n \equiv 1 \pmod 2$ admits signed product cordial labeling.

Proof. Let G be the graph obtained by duplication of all the edges of the wheel graph by vertices simultaneously. Let $V = \{v_1, v_2, \ldots, v_{3n-2}\}$ be the vertex set and the edge set be:

$$E = E_1 \cup E_2 \cup E_3 \cup E_4 \cup E_5 \cup E_6 \cup E_7$$

where: $E_1 = \{v_iv_{i+1} : 2 \le i \le n-1\}$, $E_2 = \{v_nv_2\}$, $E_3 = \{v_1v_i : 2 \le i \le n\}$, $E_4 = \{v_1v_{n+i} : 1 \le i \le n-1\}$, $E_5 = \{v_{i+1}v_{n+i} : 1 \le i \le n-1\}$, $E_6 = \{v_{i+1}v_{2n-1+i} : 1 \le i \le n-1\}$, $E_7 = \{v_{i+2}v_{2n-1+i} : 1 \le i \le n-1\}$. Here $|V(G)|$ =

$3n$-2 and $|E(G)|$ = $6n$-6. Define the vertex labeling $f : V \to \{1, -1\}$ as follows:

$f(v_1) = 1$

$$f(v_i) = \begin{cases} 1, & i \equiv 0 \pmod 2 \\ -1, & i \equiv 1 \pmod 2;\ 2 \le i \le 3n-2 \end{cases}$$

<= =>

The induced edge labeling $f^* : E(G) \to \{1, -1\}$ is defined as follows:

$$f^*(v_i v_j) = \begin{cases} 1, & \text{if } f(v_i) \text{ and } f(v_j) \text{ have same sign} \\ -1, & \text{if } f(v_i) \text{ and } f(v_j) \text{ have opposite signs} \end{cases};\ 1 \le i, j \le n-1$$

<= =>

From the above labeling pattern, we observe that the total number of vertices labeled with 1 is $\left(\frac{3n-1}{2}\right)$ and the total number of vertices labeled with -1 is $\left(\frac{3n+1}{2}\right)$. Hence $|v_f(1) - v_f(-1)|$ = 1.Similarly the total number of edges labeled with 1 is 3n-3 and the total number of edges labeled with -1 is 3n-3.Hence the total number of edges labeled with 1 and the total number of edges labeled with -1differ by zero. Hence in view of the above labeling patterns we observe that the resultant graph is a signed product cordial labeling.

Theorem 2.2: The twig graph T_m, $m \ge 3$ admits signed product cordial labeling.

Proof. Let T_m be the twig graph with $3m+2$ vertices and $3m+1$ edges, where m is the number of internal vertices. Let $V = \{v_1, v_2, \ldots, v_{3m+2}\}$ be the vertex set and the edge set be $E = E_1 \cup E_2 \cup E_3$ where $E_1 = \{v_i v_{i+1} : 1 \le i \le m+1\}$, $E_2 = \{v_i v_{m+2i-1} : 2 \le i \le m+1\}$, $E_3 = \{v_i v_{2i+m} : 2 \le i \le m+1\}$. Define the vertex labeling $f : V \to \{1, -1\}$ as follows:

Case(i): $m \equiv 1 \pmod 2$

Subcase(i): $m \equiv 1 \pmod 4$

$f(v_{m+2}) = -1$

$$f(v_i) = \begin{cases} 1, & i \equiv 1, 2 \pmod 4 \\ -1, & i \equiv 0, 3 \pmod 4;\ 1 \le i \le m+1 \end{cases}$$

$$f(v_i) = \begin{cases} 1, & i \equiv 0 \pmod 2 \\ -1, & i \equiv 1 \pmod 2;\ m+3 \le i \le 3m+2 \end{cases}$$

Subcase (ii): $m \equiv 3 \pmod 4$

$f(v_{m+2}) = 1$

$$f(v_i) = \begin{cases} 1, & i \equiv 1, 2 \pmod 4 \\ -1, & i \equiv 0, 3 \pmod 4;\ 1 \le i \le m+1 \end{cases}$$

$$f(v_i) = \begin{cases} 1, & i \equiv 0 \pmod 2 \\ -1, & i \equiv 1 \pmod 2;\ m+3 \le i \le 3m+2 \end{cases}$$

Case(ii): $m \equiv 0 \pmod 2$

Subcase (i): $m \equiv 0 \pmod 4$

$f(v_{m+1}) = 1$

$f(v_{m+2}) = -1$

$$f(v_i) = \begin{cases} 1, & i \equiv 1, 2 \pmod 4 \\ -1, & i \equiv 0, 3 \pmod 4;\ 1 \le i \le m \end{cases}$$

$$f(v_i) = \begin{cases} 1, & i \equiv 1 \pmod 2 \\ -1, & i \equiv 0 \pmod 2;\ m+3 \le i \le 3m+2 \end{cases}$$

Subcase (ii): $m \equiv 2 \pmod 4$

$$f(v_i) = \begin{cases} 1, & i \equiv 1, 2 \pmod 4 \\ -1, & i \equiv 0, 3 \pmod 4;\ 1 \le i \le m+2. \end{cases}$$

In view of the above labeling pattern, the induced edge labeling $f^* : E \to \{1, -1\}$ is defined as follows:

<= =>

$$f^*(v_i v_{i+1}) = \begin{cases} 1, & \text{if } f(v_i) \text{ and } f(v_{i+1}) \text{ have same sign} \\ -1, & \text{if } f(v_i) \text{ and } f(v_{i+1}) \text{ have opposite signs} \end{cases};\ 1 \le i \le m+1$$

$$f^*(v_i v_{m+2i-1}) = \begin{cases} 1, & \text{if } f(v_i) \text{ and } f(v_{m+2i-1}) \text{ have same sign} \\ -1, & \text{if } f(v_i) \text{ and } f(v_{m+2i-1}) \text{ have opposite signs}; 2 \le i \le m+1 \end{cases}$$

$$f^*(v_i v_{2i+m}) = \begin{cases} 1, & \text{if } f(v_i) \text{ and } f(v_{2i+m}) \text{ have same sign} \\ -1, & \text{if } f(v_i) \text{ and } f(v_{2i+m}) \text{ have opposite signs}; 2 \le i \le m+1 \end{cases}$$

In view of the above labeling pattern, the vertex and the edge labeling conditions are as follows:

<= =>

TABLE I. VERTEX AND EDGE CONDITIONS OF A TWIG GRAPH (T_M)

m	$v_f(1)$	$v_f(-1)$	$\lvert v_f(1)-v_f(-1)\rvert$
m ≡ 1 (mod 4)	(3m+3)/2	(3m+1)/2	1
m ≡ 2 (mod 4)	(3m+2)/2	(3m+2)/2	0
m ≡ 3 (mod 4)	(3m+3)/2	(3m+1)/2	1
m ≡ 0 (mod 4)	(3m+2)/2	(3m+2)/2	0
m	$e_{f^*}(1)$	$e_{f^*}(-1)$	$\lvert e_{f^*}(1)-e_{f^*}(-1)\rvert$
m ≡ 1 (mod 4)	(3m+1)/2	(3m+1)/2	0
m ≡ 2 (mod 4)	(3m+2)/2	(3m)/2	1
m ≡ 3 (mod 4)	(3m+1)/2	(3m+1)/2	0
m ≡ 0 (mod 4)	(3m)/2	(3m+2)/2	1

From the above table we observe that the difference between the edges labeled with 1 and -1 is either 1 or 0. Hence the twig graph admits signed product cordial labeling.

Theorem 2.3: A shell graph S_n, $n \ge 4$ admits signed product cordial labeling.

Proof. Let S_n, be the shell graph with n-3 chords and 2n-3 edges where $n \ge 4$.Let $V = \{v_1, v_2, \ldots, v_n\}$ be the vertex set and the edge set be $E = E_1 \cup E_2$ where $E_1 = \{v_i v_{i+1} : 1 \le i \le n-1\}$ and $E_2 = \{v_1 v_i : 3 \le i \le n\}$. Define the vertex labeling $f : V \to \{1, -1\}$ as follows:

$f(v_1) = -1$ and

$$f(v_i) = \begin{cases} 1, & i \equiv 2, 3 \pmod 4 \\ -1, & i \equiv 0, 1 \pmod 4; 2 \le i \le n. \end{cases}$$

An induced edge labeling $f^* : E \to \{1, -1\}$ is defined as:

<= =>

$$f^*(v_i v_{i+1}) = \begin{cases} 1, & \text{if } f(v_i) \text{ and } f(v_{i+1}) \text{ have same sign} \\ -1, & \text{if } f(v_i) \text{ and } f(v_{i+1}) \text{ have opposite signs}; 1 \le i \le n-1 \end{cases}$$

$$f^*(v_1 v_i) = \begin{cases} 1, & \text{if } f(v_1) \text{ and } f(v_i) \text{ have same sign} \\ -1, & \text{if } f(v_1) \text{ and } f(v_i) \text{ have opposite signs}; 3 \le i \le n \end{cases}$$

<= =>

In view of the above labeling pattern, the vertex and edge labeling conditions of a shell graph are as follows:

TABLE II. VERTEX AND EDGE CONDITIONS OF A SHELL GRAPH S_N

n	$v_f(1)$	$v_f(-1)$	$\lvert v_f(1)-v_f(-1)\rvert$
n ≡ 0 (mod 2)	n/2	n/2	0
n ≡ 1 (mod 4)	(n-1)/2	(n+1)/2	1
n ≡ 3 (mod 4)	(n+1)/2	(n-1)/2	1
n	$e_{f^*}(1)$	$e_{f^*}(-1)$	$\lvert e_{f^*}(1)-e_{f^*}(-1)\rvert$
n ≡ 0 (mod 2)	n-2	n-1	1
n ≡ 1 (mod 4)	n-1	n-2	1
n ≡ 3 (mod 4)	n-2	n-2	1

Hence the resultant shell graph admits signed product cordial labeling.

Theorem 2.4: The *Y*-tree Y_n, $n \ge 4$ admits signed product cordial labeling.

Proof. Let Y_n be a*Y*-tree with n vertices and n-1 edges. Let $V = \{v_1, v_2, \ldots, v_n\}$ be the vertex set and the edge set be $E = E_1 \cup \{v_2 v_n\}$ where $E_1 = \{v_i v_{i+1} : 1 \le i \le n-2\}$. Define the vertex labeling $f : V \to \{1, -1\}$ as follows:

$f(v_n) = -1$

$$f(v_i)=\begin{cases}1, & i\equiv 1,2 \pmod 4\\ -1, & i\equiv 0,3 \pmod 4; 1\le i\le n-1.\end{cases}$$

The induced edge labeling $f^*: E\to\{1,-1\}$ is defined as:

<= =>

$$f^*(v_iv_{i+1})=\begin{cases}1, & if\ f(v_i)\ and\ f(v_{i+1})\ have\ same\ sign\\ -1, & if\ f(v_i)\ and\ f(v_{i+1})\ have\ opposite\ signs, 1\le i\le n-2\end{cases}$$

<= =>

$f^*(v_2v_n)=-1$ since $f(v_n)=-1$ and $f(v_2)=1.$

In view of the above labeling pattern, the vertex and edge labeling conditions of a Y tree are as follows:

TABLE III. VERTEX AND EDGE CONDITIONS OF A Y TREE

n	$v_f(1)$	$v_f(-1)$	$\lvert v_f(1)-v_f(-1)\rvert$
n ≡ 1 (mod 4)	(n-1)/2	(n+1)/2	1
n ≡ 3 (mod 4)	(n+1)/2	(n-1)/2	1
n ≡ 0 (mod 2)	n/2	n/2	0
n	$e_{f^*}(1)$	$e_{f^*}(-1)$	$\lvert e_{f^*}(1)-e_{f^*}(-1)\rvert$
n ≡ 1 (mod 4)	(n-1)/2	(n-1)/2	0
n ≡ 3 (mod 4)	(n-1)/2	(n-1)/2	0
n ≡ 0 (mod 2)	(n/2)-1	n/2	1

From the above table, we observe that the Y-tree admits signed product cordial labeling.

Theorem 2.5: A web graph W(t, n), for all *t* and *n* admits signed product cordial labeling.

Proof. Let W(t, n) be a web graph with *n* vertices and *t* cycles. Let $V=\{v_1, v_2, \ldots, v_n\}$ be the vertex set and the edge set be:

$E=E_1\cup E_2\cup E_3$

Where

$E_1=\{v_iv_{i+1}: 1\le i\le nt; except\ for\ i\equiv 0 \pmod n\}$,

$E_2=\{v_{i-(n-1)}v_i: 1\le i\le 3n, i\equiv 0 \pmod n\}$,

$E_3=\{v_iv_{i+n}: 1\le i\le nt\}$. Here $|V|=n(t+1)$ and $|E|=2n+t$. Define the vertex labeling $f: V\to\{1,-1\}$ as follows:

Case (i): $t\equiv 0,1 \pmod 2;\ n\equiv 0 \pmod 2$

$$f(v_i)=\begin{cases}1, & i\equiv 1 \pmod 2\\ -1, & i\equiv 0 \pmod 2; 1\le i\le n(t+1)\end{cases}$$

Case (ii): $t\equiv 0,1 \pmod 2;\ n\equiv 1 \pmod 2$

<= =>

$$f(v_i)=\begin{cases}1, & i\equiv 1,3 \pmod n\\ -1, & i\equiv 0,2 \pmod n; 1\le i\le nt+n-1, except\ for\ i\equiv 0 \pmod n\end{cases}$$

$$f(v_i)=\begin{cases}1, & i\equiv 1 \pmod n\\ -1, & i\equiv 0 \pmod n; n\le i\le n(t+1); i\equiv 0 \pmod n\end{cases}$$

The induced edge labeling $f^*: E\to\{1,-1\}$ is defined as:

$$f^*(v_iv_{i+1})=\begin{cases}1, & if\ f(v_i)\ and\ f(v_{i+1})\ have\ same\ sign\\ -1, & if\ f(v_i)\ and\ f(v_{i+1})\ have\ opposite\ sign; 1\le i\le nt\ \ except\ for\ i\equiv 0 \pmod n\end{cases}$$

$$f^*(v_{i-(n-1)}v_i)=\begin{cases}1, & if\ f(v_i)\ and\ f(v_{i-(n-1)})\ have\ same\ sign\\ -1, & if\ f(v_i)\ and\ f(v_{i-(n-1)})\ have\ opposite\ signs, 1\le i\le 3n; i\equiv 0 \pmod n\end{cases}$$

$$f^*(v_iv_{i+n})=\begin{cases}1, & if\ f(v_i)\ and\ f(v_{i+n})\ have\ same\ sign\\ -1, & if\ f(v_i)\ and\ f(v_{i+n})\ have\ opposite\ sign; 1\le i\le nt\end{cases}$$

In view of the above labeling pattern, the vertex and the edge labeling conditions of the web graph are as follows:

TABLE IV. VERTEX AND EDGE CONDITIONS OF THE WEB GRAPH

n	t	$v_f(1)$	$v_f(-1)$	$\lvert v_f(1)-v_f(-1)\rvert$
n ≡ 0 (mod 2)	t ≡ 0 (mod 2)	n(t+1)/2	n(t+1)/2	0
n ≡ 0 (mod 2)	t ≡ 1 (mod 2)	n(t+1)/2	n(t+1)/2	0
n ≡ 1 (mod 2)	t ≡ 1 (mod 2)	n(t+1)/2	n(t+1)/2	0
n ≡ 1 (mod 2)	t ≡ 0 (mod 2)	(nt+n+1)/2	(nt+n-1)/2	1
n	t	$e_{f^*}(1)$	$e_{f^*}(-1)$	$\lvert e_{f^*}(1)-e_{f^*}(-1)\rvert$

From the above table, we observe that the difference between the number of edges labeled with 1 and -1 is 0 for the resultant graph. Hence the web graph admits signed product cordial labeling.

Theorem2.6: The graph $K_{1,m} \lozenge K_{1,n}$ for all m, n admits signed product cordial labeling except for $m \equiv 2 \pmod 4$ and $n \equiv 1 \pmod 2$..

Proof. $K_{1,m} \lozenge K_{1,n}$ is a tree obtained by taking m copies of $k_{1,n}$ and merging central vertex of each $k_{1,n}$ to each pendant vertex of $K_{1,m}$. It has totally (mn+m+1) vertices. Let $K_{1,m} \lozenge K_{1,n}$ be the graph with vertex set *V* = {v_1, v_2, …, v_{nm+m+1}}and the edge set be $E = E_1 \cup E_2 \cup E_3 \cup \ldots \cup E_{m+1}$ where:

$E_1 = \{v_1 v_i : 1 \le i \le m+1\}$,

$E_2 = \{v_2 v_{m+1+i} : 1 \le i \le n\}$

$E_3 = \{v_3 v_{m+1+i} : n+1 \le i \le 2n\}$,

$E_4 = \{v_4 v_{m+1+i} : 2n+1 \le i \le 3n\}$,

$E_5 = \{v_5 v_{m+1+i} : 3n+1 \le i \le 4n\}$, …, $E_{m+1} = \{v_{m+1} v_{m+1+i} : (m-1)n+1 \le i \le mn\}$.

Here |*V*| = *mn+m+1* and |*E*| = *mn+m*. Define the vertex labeling $f : V \to \{1,-1\}$ as follows:

Case (i): $m \equiv 0 \pmod 2; n \equiv 0 \pmod 2$ and $m \equiv 1 \pmod 2; n \equiv 0 \pmod 2$

$$f(v_i) = \begin{cases} 1, & i \equiv 1 \pmod 2 \\ -1, & i \equiv 0 \pmod 2, 1 \le i \le n(t+1) \end{cases}$$

Case (ii): $m \equiv 1 \pmod 2; n \equiv 1 \pmod 2$

$$f(v_i) = \begin{cases} 1, & i \equiv 2, 3 \pmod 4 \\ -1, & i \equiv 0, 1 \pmod 4, 1 \le i \le m+1 \end{cases}$$

and

$$f(v_i) = \begin{cases} 1, & i \equiv 1 \pmod 2 \\ -1, & i \equiv 0 \pmod 2, m+2 \le i \le mn+m+1 \end{cases}$$

Case (iii): $m \equiv 0 \pmod 4; n \equiv 1 \pmod 2$

$$f(v_i) = \begin{cases} 1, & 2 \le i \le (m/2)+1 \\ -1, & (m/2)+2 \le i \le m+1 \end{cases}$$

and

$$f(v_i) = \begin{cases} 1, & i \equiv 1 \pmod 2 \\ -1, & i \equiv 0 \pmod 2; m+2 \le i \le mn+m+1 \end{cases}$$

The induced edge labeling $f^* : E \to \{1,-1\}$ is defined as

$$f^*(v_1 v_i) = \begin{cases} 1, & if\ f(v_1)\ and\ f(v_i)\ have\ same\ sign \\ -1, & if\ f(v_1)\ and\ f(v_i)\ have\ opposite\ signs; 1 \le i \le m+1 \end{cases}$$

$$f^*(v_2 v_{m+1+i}) = \begin{cases} 1, & if\ f(v_2)\ and\ f(v_{m+1+i})\ have\ same\ sign \\ -1, & if\ f(v_2)\ and\ f(v_{m+1+i})\ have\ opposite\ signs; 1 \le i \le n \end{cases}$$

$$f^*(v_3 v_{m+1+i}) = \begin{cases} 1, & if\ f(v_3)\ and\ f(v_{m+1+i})\ have\ same\ sign \\ -1, & if\ f(v_3)\ and\ f(v_{m+1+i})\ have\ opposite\ signs; n+1 \le i \le 2n \end{cases}$$

<= =>

$$f^*(v_{m+1} v_{m+1+i}) = \begin{cases} 1, & if\ f(v_{m+1})\ and\ f(v_{m+1+i})\ have\ same\ sign \\ -1, & if\ f(v_{m+1})\ and\ f(v_{m+1+i})\ have\ opposite\ sign; (m-1)n+1 \le i \le mn \end{cases}$$

In view of the above labeling pattern, the vertex and the edge labeling conditions of the graph $K_{1,m} \Diamond K_{1,n}$ are as follows:

TABLE V. VERTEX AND EDGE CONDITIONS OF A GRAPH $K_{1,m} \Diamond K_{1,n}$

m	n	$v_f(1)$	$v_f(-1)$	$\lvert v_f(1)-v_f(-1)\rvert$
m ≡ 0 (mod 2)	n ≡ 0 (mod 2)	(mn+m+2)/2	(mn+m)/2	1
m ≡ 1 (mod 2)	n ≡ 0 (mod 2)	(mn+m+1)/2	(mn+m+1)/2	0
m ≡ 1 (mod 2)	n ≡ 1 (mod 2)	(mn+m+2)/2	(mn+m)/2	1
m ≡ 0 (mod 4)	n ≡ 1 (mod 2)	(mn+m+2)/2	(mn+m)/2	1
m	n	$e_{f^*}(1)$	$e_{f^*}(-1)$	$\lvert e_{f^*}(1)-e_{f^*}(-1)\rvert$
m ≡ 0 (mod 2)	n ≡ 0 (mod 2)	(mn+m)/2	(mn+m)/2	0
m ≡ 1 (mod 2)	n ≡ 0 (mod 2)	(mn+m-1)/2	(mn+m+1)/2	1
m ≡ 1 (mod 2)	n ≡ 1 (mod 2)	(mn+m)/2	(mn+m)/2	0
m ≡ 0 (mod 4)	n ≡ 1 (mod 2)	(mn+m)/2	(mn+m)/2	0

From the above table, we observe that the difference between the number of edges labeled with 1's and -1's is either 0 or 1. Hence the resultant graph admits signed product cordial labeling.

Observation 2.7: The line graph of a path graph is again a path graph which admits signed product cordial labeling.

Observation 2.8: The Petersen graph is a signed product cordial graph.

III. CONCLUSION

In our present study, an existence of signed product cordial labeling has been examined for some classes of graphs. Analyzing the properties and applications of signed product cordial labeling are our future work.

REFERENCES

[1] Barrientos C, Graceful Arbitrary Super Subdivisions of Graphs, Indian Journal of Pure and Applied Mathematics, 2007; 38, 445-450.

[2] Baskar Babujee J and Shobana L, On Signed Product Cordial Labeling, Applied Mathematics, 2011; 12, 1525-1530.

[3] Deb P and Limaye N.B, On Harmonious Labeling of Some Cycle Related Graphs, Ars.Combin.,2002; 65, 177-197.

[4] J.A. Gallian, "A Dynamic Survey of Graph Labeling", Electronic Journal of Combinatorics, #DS6, (2017), pp. 1–432.

[5] Kang Q.D, Liang Z.H,Gao Y. Zand Yang G.H, On the Labeling of Some Graphs, J. Combin Math Combin Comput., 1996; 22, 193-210.

[6] Selvam B, ThirusanguK &Ulaganathan P. P, Graceful and Skolem Graceful Labeling Extended Duplicate Twig Graphs, International Journal of Combinatorial Graph Theory and Applications, 2011; 4, 141-149.

[7] Shobana L,BaskarBabujee J, Signed Product Cordial Labeling on Special Graphs Global Journal of Pure and Applied Mathematics, 2016; 12, 376-380.

[8] Vaidya S. Kand Dani N. A, Cordial and 3-equitable Graphs Induced by Duplication of Edge, Math.Today, 2011; 27, 71-82.