Mathematical Modelling of Discrete Time Bandlimited Signals for Digital Communication

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Abstract - **In this paper we use Nyquist-Shannon sampling theorem for bandlimited signals to propose and verify new techniques to construct and analyze: i) Discrete Time models for Continuous Time signals, and ii) Continuous Time models for Discrete Time signals. These reconstruction and processing techniques include some of the most practical mathematical issues in Discrete Time signal/system research area such as digital signal processing and digital communication in the last two decades. Detailed mathematical descriptions and figures are used to illustrate the concepts. Furthermore, we propose and successfully verify the application of Discrete Time reconstruction and processing techniques to Super Resolution Reconstruction (SRR) to model higher resolution signals by using DT sampled signal.**

Keywords - Continuous-Time Fourier Transform (CT-FT), Discrete-Time Fourier Transform (DT-FT), Aliasing Problem, Digital Signal Processing (DSP).

I. INTRODUCTION

Due to a large requirement of both digital signals (which are the digitized Discrete-Time (DT) signals) and Discrete-Time (DT) signals [1, 5, 10] (such as speech, image [6, 7, 8], communication data [11], etc.), in order to implement and research of the modern algorithms and mathematical techniques for applying on modern electronic devices based on microprocessors / microcontrollers (such as smart phone, PDA and digital camera, etc.), the real observed signals, which usually are Continuous-Time (CT) signals, can be converted into Discrete-Time (DT) signals by using any mathematical sampling techniques [2, 4]: a uniform sampling with various parametric basis expansion modeling [3, 12] or a non-uniform sampling, which is based on the mathematical sampling concept of Discrete-Time (DT) analysis of Aliasing and Non-Aliasing for periodic signals [9].

II. MATHEMATICAL MODEL OF BANDLIMITED SIGNAL RECONSTRUCTION

Due to the sampling theorem [9], a sequence of a sampled signal, which is acquired from the continuous-time (CT) bandlimited signal [2, 4] by taking frequently enough, can be used for absolutely representing the original CT signal [1, 5, 10] or for reconstructing the original CT signal with sampling period information. Hence, the sequence of an impulse train $x_s(t)$ can be mathematically defined in the term of a sampled signal $x[n]$ as following:

$$x_s(t) = \sum_{n=-\infty}^{\infty} x[n]\delta(t - nT) \qquad (1)$$

where T is the sampling period and $\delta(\cdot)$ is the delta impulse function [3, 12].

Assume that the above impulse train $x_s(t)$ is the input signal of the ideal low-pass CT filter [3, 12], which has the system function (so called the frequency response) $H_r(j\Omega)$ or the impulse response $h_r(t)$ that can be mathematically, defined as $h_r(t) = \sin(\pi t/T)/(\pi t/T)$ or:

$$h_r(t - nT) = \sin(\pi(t - nT)/T)/(\pi(t - nT)/T)$$

therefore, the output of the ideal low-pass CT filter [1,9] can be mathematically defined as:

$$x_r(t) = \sum_{n=-\infty}^{\infty} x[n]h_r(t - nT) \rightarrow \; x_r(t) = \sum_{n=-\infty}^{\infty} x[n]\frac{\sin(\pi(t - nT)/T)}{\pi(t - nT)/T} \qquad (2)$$

From the above equation, the CT reconstructed signal $x_r(t)$ can be mathematically expressed in the a basis function form of a linear combination of $h_r(t - nT)$ and the sampling signal

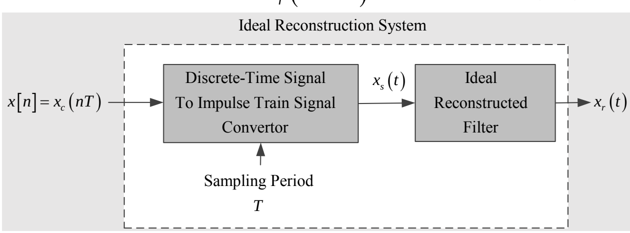

Figure 1(a) Block diagram of ideal reconstruction system for the bandlimited signal by using the ideal low-pass filter.

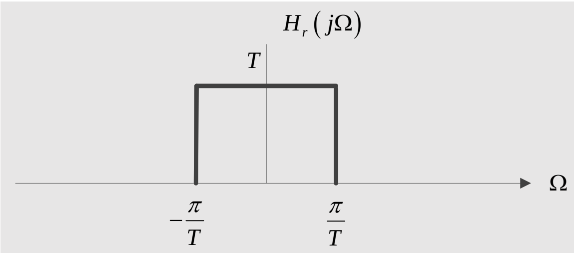

Figure 1(b) The system function (the frequency response) $H_r(j\Omega)$ of the ideal low-pass filter

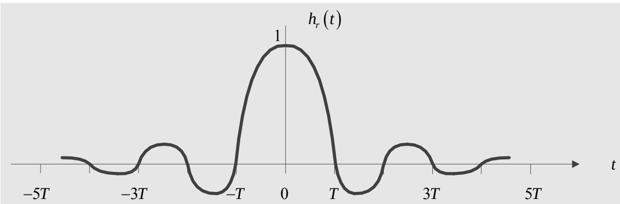

Figure 1(c) The impulse response $h_r(t)$ of the ideal low-pass filter

$x[n]$. The block diagram of ideal reconstruction system for the bandlimited signal by using the ideal low-pass filter, the system function (so called the frequency response) $H_r(j\Omega)$ of the ideal low-pass filter and the impulse response $h_r(t)$ of the ideal low-pass filter can be illustrated in the figure 1(a), 1(b) and 1(c), respectively.

From the mathematical tutorial of DT analysis [9], if the sampling signal is defined as $x[n]=x_c(nT)$ and the original CT signal is a bandlimited signal ($X_c(j\Omega)=0$ for $|\Omega|\geq\pi/T$) then the reconstruction signal is defined as $x_r(t)=x_c(t)$.

From the definition of the ideal low-pass filter impulse response with $t=nT$, the impulse response $h_r(t)=\sin(\pi t/T)/(\pi t/T)$ can be mathematically simplified as following:

$$h_r(t)=\sin(\pi t/T)/(\pi t/T)$$
$$\rightarrow \quad h_r(nT)=\sin(\pi nT/T)/(\pi nT/T) \tag{3a}$$
$$\rightarrow \quad h_r(nT)=\sin(\pi n)/(\pi n) \tag{3b}$$

From l'Hopital's rule [3, 12], the impulse response $h_r(t)$ of the ideal low-pass filter can be mathematically simplified as following:

$$h_r(t)=\sin(\pi t/T)/(\pi t/T)$$
$$\rightarrow \quad h_r(0)=1 \tag{3c}$$
$$\rightarrow \quad h_r(nT)=0 \quad \text{,for } n=\pm1,\pm2,\pm3,\ldots \tag{3d}$$

Hence, if $x[n]=x_c(nT)$ then $x_r(mT)=x_c(mT)$ for all integer value of m.

The case study of CT original signal $x_c(t)$, the sequence of an impulse train $x_s(t)$ (,which is sampled from $x_c(t)$) and the reconstructed CT signal $x_r(t)$ (which is defined in the form of $x[n](\sin(\pi(t-nT)/T)/\pi(t-nT)/T)$ in Eq. (2)) can be illustrated in the figure 2(a), 2(b) and 2(d), respectively.

From the above idea of the bandlimited signal reconstruction process, the ideal Discrete-To-Continuous-Time (D/C) converter can be illustrated in the following figure. The ideal converter consists of two main process: the DT signal to impulse train signal converter (which is mathematically defined in Eq.(1)) and the ideal reconstructed filter (which is mathematically defined in Eq.(2)).

From the frequency domain prospective, the Fourier transform of the reconstructed signal $x_r(t)$ (in Eq.(2)) can be mathematically defined as following:

$$x_r(t)=\sum_{n=-\infty}^{\infty}x[n]h_r(t-nT)$$
$$\overset{CTFT}{\rightarrow} \quad X_r(j\Omega)=\sum_{n=-\infty}^{\infty}x[n]H_r(j\Omega)e^{-j\Omega Tn} \tag{4a}$$
$$\rightarrow \quad X_r(j\Omega)=H_r(j\Omega)X(e^{-j\Omega T}) \tag{4b}$$

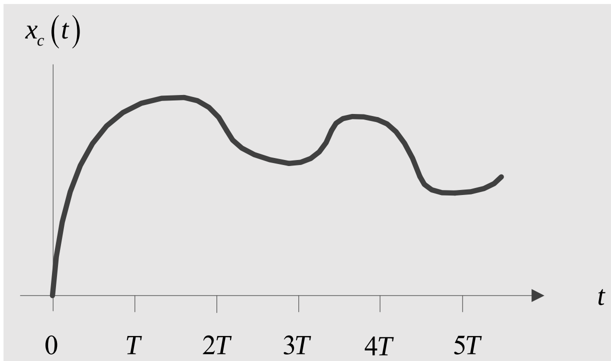

Figure 2 (a) The case study of the CT original signal $x_c(t)$

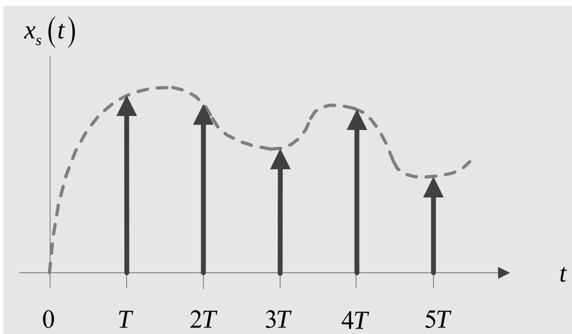

Figure 2 (b) the case study of the sequence of an impulse train $x_s(t)$

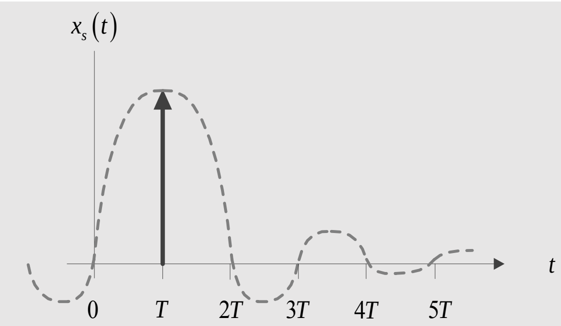

Figure 2 (c) The case study of the one of an impulse train $x_s(t)$

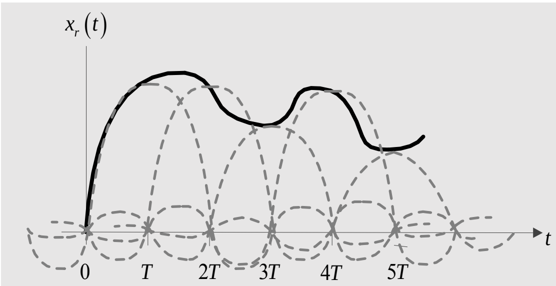

Figure 2 (d) its basis function and the case study of the reconstructed CT signal $x_r(t)$

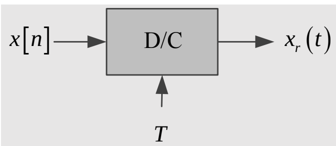

Figure 3 (a) The equivalent process of an ideal D/C converter

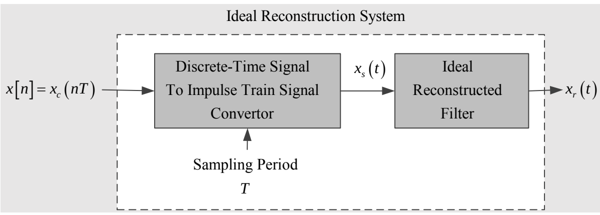

Figure 3 (b) The process of an ideal D/C converter for a bandlimited signal reconstruction

From the above equation of the ideal bandlimited signal reconstruction process, the mathematical term of $X\left(e^{-j\omega}\right)$ is frequency scaled where $\omega = \Omega T$. Consequently, the ideal reconstructed filter $H_r(j\Omega)$ chooses the optimal reconstructed period T (as illustrated in Fig 3(b)) for compensating the scaling effect due to the sampling process therefore the optimal reconstructed period is generally set to be equal to the sampling period. If the DT sampled signal $x[n]$ has been acquired from a bandlimited signal by sampling process at the Nyquist rate [4,8] then the reconstructed DT signal $x_r(t)$ can be perfectly recovered and is identical with the original CT bandlimited signal $x_c(t)$.

III. MATHEMATICAL MODEL OF DT PROCESSING FOR CT SIGNALS

For analyzing and processing CT signals, the DT systems [1,9], which are implemented by microprocessors or computers, are usually designed for these proposes and the overall process of this system can be illustrated in the following figure. From figure 4(a), the overall process of this system is identical with the CT process because both system input ($x_c(t)$) and system output ($y_r(t)$) are CT signals therefore this overall system properties is bank on both the chosen DT system and the chosen sampling rate (T).

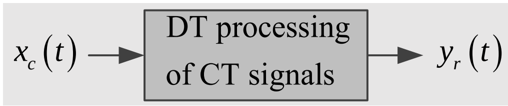

Figure 4 (a) The equivalent process of a DT process for a CT signal

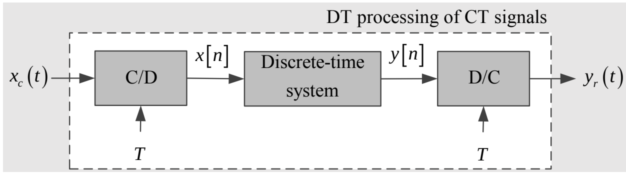

Figure 4 (b) The detail structure of a DT process for a CT signal

From the concept of both a C/D converter and D/C converter (which is descripted in the proceeding section as illustrated in figure 4(b)), the relationship between a CT input signal $x_c(nT)$ and a DT input signal $x[n]$ by using the C/D converter can be mathematically defined as following.

$$x[n] = x_c(nT)$$

$$\overset{FT}{\rightarrow} \qquad X\left(e^{j\omega}\right) = \frac{1}{T}\sum_{k=-\infty}^{\infty} X_c\left[j\left(\frac{\omega}{T} - \frac{2\pi k}{T}\right)\right] \tag{5}$$

From the concept of a D/C converter, the relationship between a DT output signal ($y[n]$) and a CT output signal ($y_r(t)$) by using the D/C converter can be mathematically defined as following:

$$y_r(t) = \sum_{n=-\infty}^{\infty} y[n]\left(\sin\left(\pi(t-nT)/T\right)\big/\pi(t-nT)/T\right)$$

$$\overset{FT}{\rightarrow} \; Y_r(j\Omega) = H_r(j\Omega)Y\left(e^{j\Omega T}\right) = \begin{cases} TY\left(e^{j\Omega T}\right) & |\Omega| < \pi/T \\ 0 & \text{otherwise} \end{cases} \tag{6}$$

In general, if the DT system is nonlinear and time varying [1,5,9] then the mathematical modeling of the input-

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output relationship by using Fourier transform is considerably so complex therefore the DT system is usually defined to be a linear and time invariance system (or LTI system), which is mathematically explained in detail as following section.

A. Mathematical Model of DT-LTI Process Concept for CT Signal

For a DT-LTI system with both a sampling period T and a reconstructed period T as illustrated in figure 4(b), the relationship between a DT input signal ($x[n]$ or $X(e^{j\omega})$), impulse response of the DT system ($h[n]$ or $H(e^{j\omega})$) and a DT output signal ($y[n]$ or $Y(e^{j\omega})$) can be mathematically defined as following:

$$y[n] = h[n] * x[n]$$

$$\overset{DTFT}{\rightarrow} \quad Y(e^{j\omega}) = H(e^{j\omega}) X(e^{j\omega}) \quad (7a)$$

$$\rightarrow \quad Y(e^{j\Omega T}) = H(e^{j\Omega T}) X(e^{j\Omega T}) \quad (7b)$$

For an overall DT system as illustrated in figure 4(b), the frequency domain relationship between a DT input signal ($x[n]$ or $X(e^{j\omega})$) and a CT output signal ($y_r(t)$ or $Y_r(j\Omega)$) can be mathematically defined as following:

$$Y_r(j\Omega) = H_r(j\Omega) Y(e^{j\Omega T})$$

$$\rightarrow \quad Y_r(j\Omega) = H_r(j\Omega) H(e^{j\Omega T}) X(e^{j\Omega T})$$

$$\rightarrow Y_r(j\Omega) = H_r(j\Omega) H(e^{j\Omega T}) \left(\frac{1}{T} \sum_{k=-\infty}^{\infty} X_c\left[j\left(\Omega - \frac{2\pi k}{T} \right) \right] \right) \quad (8a)$$

For the continuous-time (CT) bandlimited input signal, which is $X_c(j\Omega) = 0$ for $|\Omega| \geq \pi/T$, the $H_r(j\Omega)$ will abolish the factor $1/T$ (which is the result from the sampling process) and chooses the mathematical term in the above equation for $k = 0$ as following:

$$\rightarrow \quad Y_r(j\Omega) = \begin{cases} H_r(j\Omega) X_c(j\Omega) & |\Omega| < \pi/T \\ 0 & |\Omega| \geq \pi/T \end{cases} \quad (8b)$$

Consequently, if the continuous-time (CT) bandlimited input signal, which is sampled at above the Nyquist rate [8], then the CT output signal can be mathematically defined as following:

$$\rightarrow \quad Y_r(j\Omega) = H_{eff}(j\Omega) X_c(j\Omega)$$

where $H_{eff}(j\Omega) = \begin{cases} H_r(e^{j\Omega T}) & |\Omega| < \pi/T \\ 0 & |\Omega| \geq \pi/T \end{cases}$ (9)

From the equivalent process of a DT- LTI process for a CT signal as illustrated in the figure 4(a), the equivalent effective transfer function can be mathematically defined as the above equation.

A1. Mathematical Model Case Study of DT-LTI process of Lowpass Filter.

Analysis the DT-LTI lowpass filter process in the frequency domain, which has the frequency response $H(j\Omega) = 1$ for $|\omega| < \omega_c$ and 0 for $\omega_c < |\omega| \leq \pi$, when the input signal is a continuous-time (CT) bandlimited signal (where $\Omega_N T < \omega_c$) [2, 4]

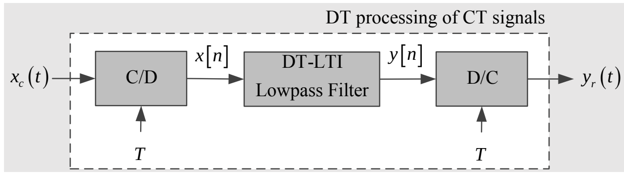

Figure 5 (a) The DT-LTI process of Lowpass filter for a CT signal

From this frequency domain analysis [4], the CT input signal, the transfer function of DT-LTI Lowpass filter and the CT output signal can be illustrated in the figure 5(d), figure 5(c) and figure 5(i), respectively therefore even through the lowpass filter is the DT system, the system processes as the CT system.

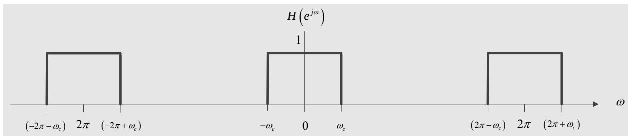

Figure 5 (b) The frequency response of a DT-LTI process of Lowpass filter (from DTFT prospective)

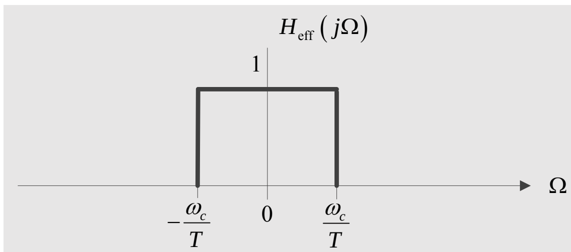

Figure 5 (c) The overall/effective frequency response of a DT process (from CTFT prospective)

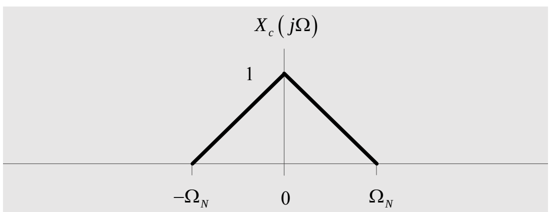

Figure 5 (d) The FT of a continuous-time (CT) bandlimited input signal

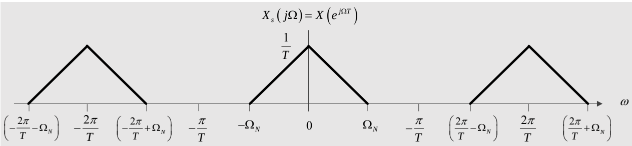

Figure 5 (e) The FT of the sampling signal of a CT bandlimited input signal

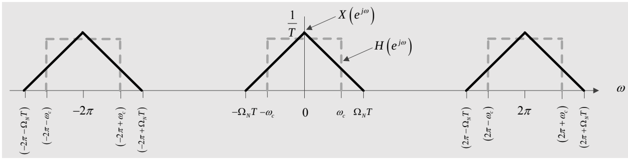

Figure 5 (f) The FT of the DT sampling signal and the frequency response of a DT-LTI process

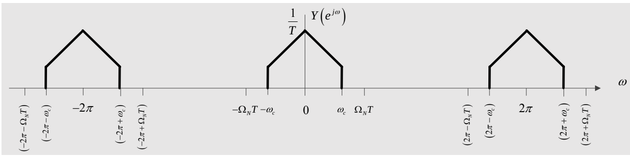

Figure 5 (g) The FT of the DT output signal from the DT-LTI process

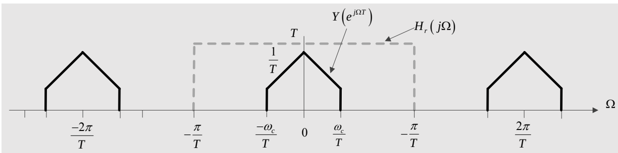

Figure 5 (h) The FT of the DT output signal and the frequency response of the ideal reconstructed filter

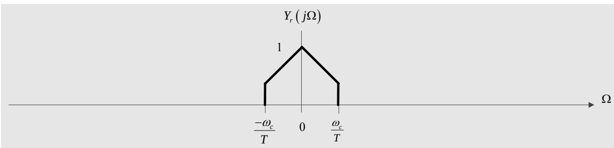

Figure 5 (i) The FT of the CT output signal

A2. Mathematical Model Case study of DT-LTI process of Differentiator.

Analysis the following DT-LTI Differentiator process in the frequency domain [2, 4], which is defined by $y_c(t) = d(x_c(t))/dt$ or has the frequency response $H(j\Omega) = j\Omega$, when the input signal is a continuous-time (CT) bandlimited signal(where $\Omega_N T < \omega_c$)

$$H(j\Omega) = j\Omega \;\rightarrow\; H_{eff}(j\Omega) = \begin{cases} j\Omega & |\Omega| < \pi/T \\ 0 & |\Omega| \geq \pi/T \end{cases} \quad (10)$$

Therefore, the frequency response of the DT system can be mathematically defined as following:

$$H(e^{j\omega}) = \frac{j\omega}{T}, |\omega| < \pi \;\xrightarrow{IFT}\; h[n] = \int_{-\pi}^{\pi}\left(\frac{j\omega}{T}\right)e^{j\omega n}d\omega \quad (11a)$$

$$\rightarrow h[n] = \frac{\pi n\cos\pi n - \sin\pi n,}{\pi n^2 T} \quad ,-\infty < n < \infty \quad (11b)$$

$$\rightarrow h[n] = \begin{cases} 0 & n = 0 \\ \dfrac{\cos\pi n}{nT} & n \neq 0 \end{cases} \quad (11c)$$

The overall frequency response of CT differentiator is illustrated as figure 6(a) and 6(b) and the overall frequency response of DT differentiator is illustrated as figure 6(c) and 6(d).

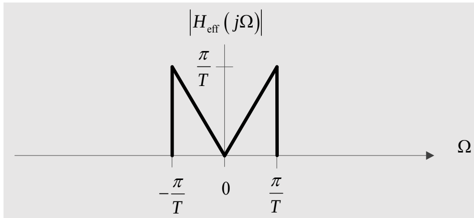

Figure 6 (a) The overall/effective frequency response of a CT Differentiator ($H_{eff}(j\Omega)$)

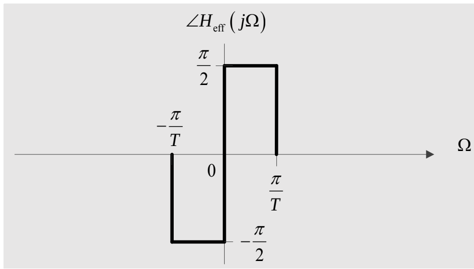

Figure 6 (b) The overall/effective frequency response of a CT Differentiator ($H_{eff}(j\Omega)$)

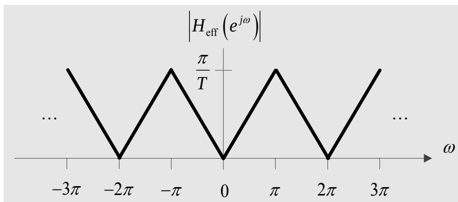

Figure 6 (c) The overall/effective frequency response of a DT Differentiator ($H(e^{j\omega})$)

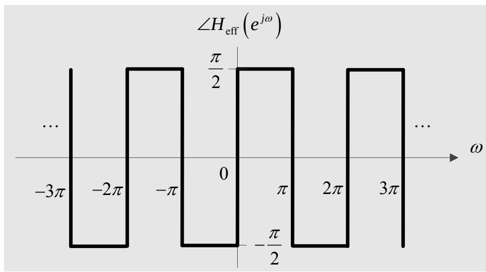

Figure 6 (d) The overall/effective frequency response of a DT Differentiator ($H(e^{j\omega})$)

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B. Mathematical Model of Impulse Invariance Concept

Given a CT system with the effective transfer function $H_{eff}(j\Omega)$ for the continuous-time (CT) bandlimited input signal (where $\Omega_N T < \omega_c$) [2, 4], the DT-LTI system with $H(e^{j\omega}) = H_c(j\omega/T), |\omega| < \pi$ is implemented in the form of figure 7.

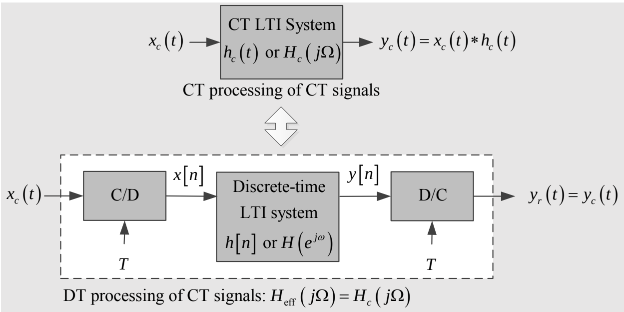

Figure 7 The DT-LTI system of CT signals and its equivalent system

Due to the $H_c(j\Omega) = 0, |\Omega| \ge \pi/T$ from continuous-time (CT) bandlimited input signal property, the mathematical relationship between the CT impulse response and the DT impulse response can be mathematically defined as following:

$$h[n] = Th_c(nT) \xrightarrow{FT} H(e^{j\omega}) = \frac{1}{T}\sum_{k=-\infty}^{\infty} H_c\left[j\left(\frac{\omega}{T} - \frac{2\pi k}{T}\right)\right] \quad (12a)$$

$$\rightarrow H(e^{j\omega}) = \frac{1}{T} H_c\left(j\frac{\omega}{T}\right) \quad , |\omega| < \pi \quad (12b)$$

B1. Mathematical Model Case study of Impulse Invariance calculation of DT lowpass filter process.

Determine the impulse response and its frequency response of the DT-LTI lowpass system with cutoff frequency $\omega_c < \pi$.

From $\omega_c < \pi$, $\Omega_c = (\omega_c/T) < (\pi/T)$ and the frequency response of the CT system can be mathematically defined as following:

$$H_c(j\Omega) = \begin{cases} 1 & |\Omega| < \Omega_c \\ 0 & |\Omega| \ge \Omega_c \end{cases} \xrightarrow{IFT} h_c(t) = \frac{\sin(\Omega_c t)}{\pi t} \quad (13)$$

From the impulse invariance concept, the impulse response and its frequency response of DT-LTI Lowpass system can be mathematically defined as following:

$$h[n] = Th_c(nT)$$

$$\rightarrow \quad h[n] = T\sin(\Omega_c nT)/(\pi nT) \quad (14a)$$

$$\rightarrow \quad h[n] = \sin(\omega_c n)/(\pi n) \quad (14b)$$

$$\xrightarrow{FT} \quad H(e^{j\omega}) = \begin{cases} 1 & |\omega| < \omega_c \\ 0 & \omega_c \le |\omega| \le \pi \end{cases} \quad \text{where } \omega_c = \Omega_c T \quad (14c)$$

C. Mathematical Model of CT Process Concept for DT Signal

For complementary prospective of the DT system for CT signals (in the proceeding section), the CT system for DT signals is mathematically analyzed, which can be illustrated in the following figure, in order to make the reader completely understanding of the concept of DT signals and its sampling process.

Due to the continuous-time (CT) bandlimited input signal property for the ideal D/C converter (by using the ideal lowpass filter), both $X_c(j\Omega) = 0, |\Omega| \ge \pi/T$ and $Y_c(j\Omega) = 0, |\Omega| \ge \pi/T$ then there is no aliasing problem in the sampling process and the CT input $x_c(t) = 0$ and the CT output $y_c(t) = 0$ can be mathematically defined as following:

$$x_c(t) = \sum_{n=-\infty}^{\infty} x[n] \frac{\sin(\pi(t - nT)/T)}{\pi(t - nT)/T} \text{ where } x[n] = x_c(nT) \quad (15a)$$

$$y_c(t) = \sum_{n=-\infty}^{\infty} y[n] \frac{\sin(\pi(t - nT)/T)}{\pi(t - nT)/T} \text{ where } y[n] = y_c(nT) \quad (15b)$$

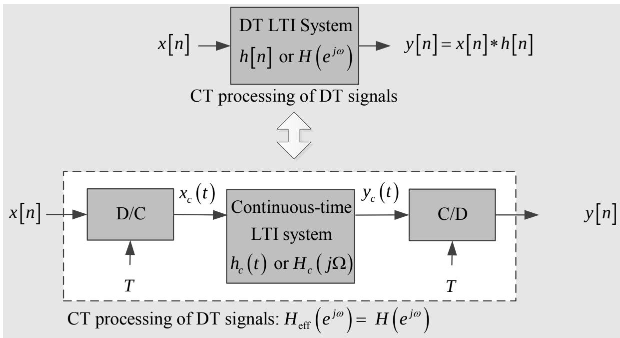

Figure 8 The CT-LTI system of DT signals and its equivalent system

By using the FT, the mathematical relationship of the signals in the CT-LTI system of DT signals (shown in figure 8) can be mathematically defined as following:

$$X_c(j\Omega) = TX(e^{j\Omega T}) \quad , |\Omega| < \pi/T \quad (16a)$$

$$Y_c(j\Omega) = H_c(j\Omega) X_c(j\Omega) \quad (16b)$$

$$Y_c(e^{j\omega}) = \frac{1}{T} Y_c\left(j\frac{\omega}{T}\right) \quad , |\omega| < \pi \quad (16c)$$

The overall frequency response of the CT-LTI system of DT signals (shown in figure 8) can be mathematically defined as following:

$$H_c\left(e^{j\omega}\right) = H_c\left(j\frac{\omega}{T}\right) \quad ,|\omega| < \pi \tag{17}$$

where $H_c(j\Omega) = H_c\left(e^{j\Omega T}\right), |\Omega| < \pi/T$

C1. Mathematical Model Case study of CT-LTI process of Non-Integer Delay for DT signals.

This section analyzes the impulse response and its frequency response of the CT-LTI process of Non-Integer Delay [2, 4] for DT signals.

If Δ is defined as a non-integer then the mathematical relationship of the DT input signal $x[n]$ and the DT output signal $y[n]$ can be mathematically defined as following:

$$y[n] = x[n-\Delta] \xrightarrow{FT} H\left(e^{j\omega}\right) = e^{-j\omega\Delta} \quad ,|\omega| < \pi \tag{18}$$

The DT output signal $y[n]$ can be mathematically defined as following:

$$y[n] = y_c[nT] = x_c(nT - T\Delta)$$

$$\rightarrow \quad y[n] = \sum_{n=-\infty}^{\infty} x[k] \left.\frac{\sin\left(\pi(t - T\Delta - kT)/T\right)}{\pi(t - T\Delta - kT)/T}\right|_{t=nT}$$

$$\rightarrow \quad y[n] = \sum_{n=-\infty}^{\infty} x[k] \frac{\sin\left(\pi(n-k-\Delta)\right)}{\left(\pi(n-k-\Delta)\right)} \tag{19}$$

From the convolution definition of $y[n] = x[n] * h[n]$, the impulse response of the DT system can be mathematically defined as $h[n] = \frac{\sin\left(\pi(n-k-\Delta)\right)}{\left(\pi(n-k-\Delta)\right)}, -\infty < n < \infty$.

If Δ is defined as an integer ($\Delta = n_0$) then the above equation can be mathematically simplified as $h[n] = \delta[n - n_0]$ but if Δ is defined as a non-integer ($\Delta = T/2$) then the relationship of the DT input signal $x[n]$ and the case study of a DT output signal $y[n]$ is defined as Eq.(19) and can be illustrated as following figure.

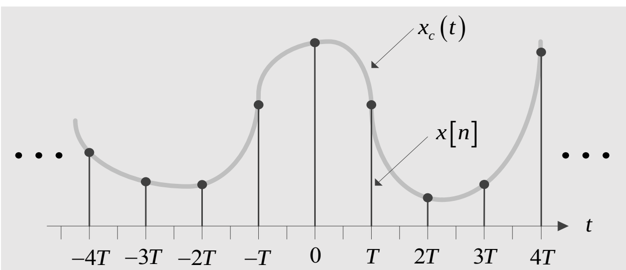

Figure 9(a) The case study of the relationship of the DT input signal $x[n]$ and the CT input signal $x_c(t)$ for CT-LTI system

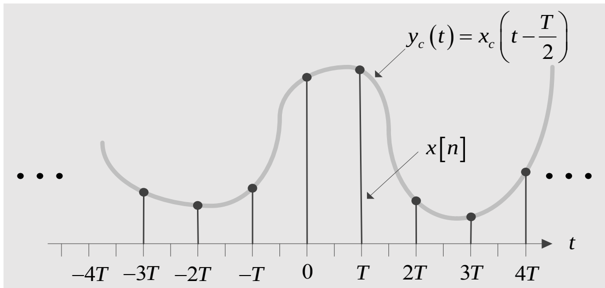

Figure 9(b) The case study of the relationship of the DT input signal $x[n]$ and the CT input signal $x_c(t)$ for CT-LTI system of half-sample delay ($\Delta = T/2$).

C2. Mathematical Model Case study of DT-LTI process of Non-Integer Moving-Average for DT signals.

Given the impulse response and its frequency response of the DT-LTI Non-Integer Moving-Average with frequency response can be mathematically defined as $H\left(e^{j\omega}\right) = (M+1)^{-1}\left(\sin\left(\omega(M+1)/2\right)/\sin(\omega/2)\right)e^{j\omega M/2}, |\omega| < \pi$ (as illustrated in figure 10(a))

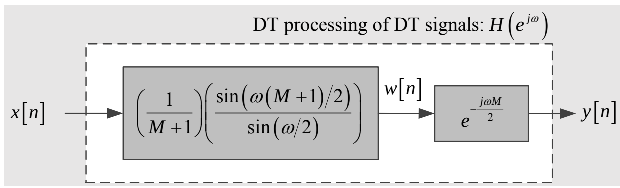

Figure 10 (a) The DT-LTI Non-Integer Moving-Average system of DT signals.

If $M = 5$ for the moving-average system [2, 4] and the DT input signal can be mathematically defined as $x[n] = \cos(0.25\pi n)$ (as illustrated in figure 10(b)) then determine the DT output signal (as illustrated in figure 10(c))

The DT output signal $y[n]$ can be mathematically defined as following:

$$y[n] = H\left(e^{j0.25\pi}\right)\frac{1}{2}e^{j0.25\pi n} + H\left(e^{-j0.25\pi}\right)\frac{1}{2}e^{-j0.25\pi n} \tag{20a}$$

$$\rightarrow \quad y[n] = \begin{cases} \frac{1}{2}\frac{\sin\left[3(0.25\pi)\right]}{6\sin(0.125\pi)} e^{-j(0.25\pi)\frac{5}{2}} e^{j0.25\pi n} \\ +\frac{1}{2}\frac{\sin\left[3(-0.25\pi)\right]}{6\sin(-0.125\pi)} e^{j(0.25\pi)\frac{5}{2}} e^{-j0.25\pi n} \end{cases} \tag{20b}$$

$$\rightarrow \quad y[n] = 0.308\cos\left[0.25\pi(n-2.5)\right] \tag{20c}$$

From the result, this moving-average system with $M = 5$ scales down the amplitude of the input cosine signal and proposes a phase shifting at 2.5 samples delay (as illustrated in figure 10(c)).

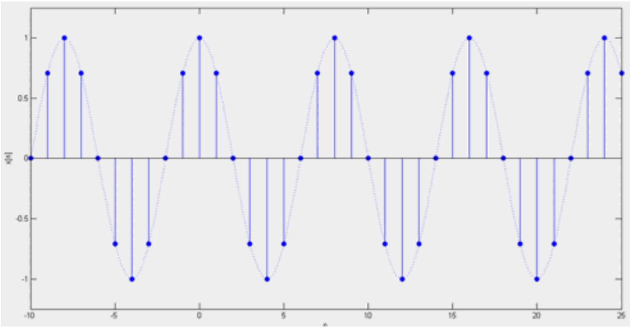

Figure 10 (b) The DT input signal $x[n] = \cos(0.25\pi n)$

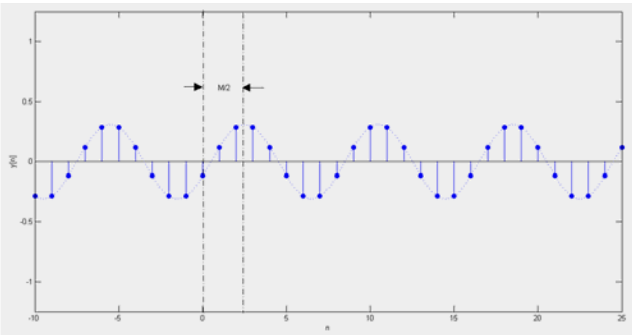

Figure 10 (c) The DT output signal $y[n] = 0.308\cos[0.25\pi(n-2.5)]$ of the system.

V. CONCLUSION

We proposed and successfully verified techniques to construct and analyze: i) Discrete Time models for Continuous Time signals, and ii) Continuous Time models for Discrete Time signals, for the general case of bandlimited signals to form the mathematical foundation for future work. These reconstruction and processing techniques include some of the most practical mathematical issues in the Discrete Time signal/system research area such as Digital Signal Processing (DSP) [2, 4] and digital communication [11] in the last two decades. Detailed mathematical descriptions and figures were used to illustrate the concepts.

Furthermore, we proposed and successfully verified the application of DT reconstruction and processing techniques to Super Resolution Reconstruction (SRR) [6,7] for reconstructing higher resolution signals by using DT sampled signal.

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