Reliable Stabilization in T-S Fuzzy Feedback Control Systems through Adaptive Lyapunov Function

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Abstract - **In industry, most of the physical systems are nonlinear with different forms. It is often difficult for design and control the nonlinear system. The non-linear systems with dynamic disturbances, which are caused due to modeling errors, measurement noise, external/ internal disturbances, delayed input, etc., will affect the system. To conquer this problem, T-S fuzzy model is designed for efficient management in the control of complex systems. In this paper, we propose a Reliable Takagi-Sugeno fuzzy controller via state feedback and output feedback controller to provide robust stabilization with parameter uncertainty and disturbances. The motivation behind this fuzzy dynamic output feedback controller design guarantees the robust asymptotic stability of the closed-loop system and guarantees a sufficient constraint on disturbance attenuation for every single admissible uncertainty in which the constraints are based on the quadratic affine Lyapunov function approach, which is less conservative than the common Lyapunov function. The stability functions are expressed in the form of Linear Matrix Inequalities. A trial result is abused to illustrate the effectiveness and possibilities of the proposed approach.**

Keywords - T-S fuzzy; State feedback and output feedback; closed-loop system; quadratic affine Lyapunov.

I. INTRODUCTION

A control system can oversee or direct the conduct of alternate systems. There should be a clear mathematical relation between the input and the output in control system [1]. In practice, there exist uncertainty in plants due to non-linearity and disturbances. In order to comprehend uncertainty a few control strategies have been created. Fuzzy logic hypothesis is one of the control techniques and it is the efficient approach to manage the uncertainty issue for complex nonlinear system [2]. A fuzzy controller utilizes fuzzy standards such as though then proclamations including fuzzy logic, fuzzy sets and fuzzy interference. One of the fuzzy systems is Takagi-Sugeno (T-S) fuzzy model is an effective device in approximating most complex non-linear system. The T-S models the non-linear system by weighted whole of linear time invariant systems [3]. It is the combination of linear models connected by scalar enrollment function to describe the nonlinear system. It can represent an arrangement of complex and non-linear systems by fuzzy thinking and fuzzy sets [4]. The principle advantage of T-S fuzzy system is the simplification, which facilitates for advance examination and union of the complex non-linear system [5, 6].

The application that utilizations fuzzy models are warming, aerating and cooling system, ventilating, robot, warm exchange pilot plant and wheel inverted pendulum [7]. In these areas, the correct selection of fuzzy tenets is important in T-S Fuzzy model to control the system, which gives strength and nature of the system. T-S model can accurately rough the non-linear system and the steadiness; control will be investigated [8, 9]. The primary motivation behind control system is to orchestrate the controller to guarantee the system security when all components are in operation as well as the system under flauts [10]. In T-S fuzzy model it contains bring down enrollment functions and upper participation functions to deal with the uncertainty in the enrollment function. The uncertain parameter of enrollment function consists of outside disturbances, modeling error and complex system deviation. So the participation function should be appropriately decided [11]. The strength conditions for T-S fuzzy control for a nonlinear plant depend on the Lyapunov functions. The Lyapunov function guarantees that the signals in the closed circle system are guaranteed to be limited [12].

Motivated by the above discussion, the contribution of this paper is to ensure the robust asymptotic stability of the closed-loop system with the help of Reliable Takagi-Sugeno Fuzzy controller via state feedback and output feedback controller with parameters uncertainty and disturbances. Moreover, this fuzzy dynamic output feedback controller design guarantees the robust asymptotic stability of the closed-loop system and assurances an adequate constraint on disturbance attenuation for every single admissible uncertainty in which the constraints are based on the quadratic affine Lyapunov function approach, which is less conservative than the common Lyapunov function.

This paper is organized as follows. Section 2 describes about the related researches. Section 3 discusses the issue formulation of T-S fuzzy system. Section 4 illustrates the approved results. Section 5 describes the conclusion and Section 6 includes the references for our work.

II. RELATED RESEARCHES

Hongyi et al. [13] have actualized interim type 2 (IT2) T-S fuzzy systems with time-fluctuating delays and exogenous disturbances to manage the ideal guaranteed cost sliding mode control issue. An adaptive technique is exhibited to deal with the time-fluctuating weight coefficients reflecting the change of the uncertain parameters in the presence of the uncertain parameters covered up in participation functions. In view of the system output, another indispensable sliding surface was exhibited. An adaptive sliding mode controller are utilized to compensate the system perturbation or modeling error and the reachability of the sliding surface can be guaranteed with a definitive uniform boundedness of the closed-circle system. For the resulting time-delay control system, ideal conditions of a H_2 guaranteed cost function and a H_∞ performance list are built up. An upset pendulum system represented by IT2 fuzzy model illustrated the advantages and effectiveness of the control scheme.

Jun et al. [14] have introduced two autonomous Bernoulli distributions to describe the arbitrarily occurring uncertainties and pick up fluctuations all the while in strong limited time boundedness of a class of Takagi– Sugeno (T–S) fuzzy stochastic systems. A state feedback fuzzy controller was designed in such a way that the closed-circle fuzzy system is limited time stochastic limited with a prescribed H_∞ performance. The sufficient conditions on the existence of the finite time H_∞ controller are determined. A numerical case was abused to illustrate the effectiveness and possibilities of this approach.

Shen et al. [15] have introduced an approximated-based adaptive fuzzy control approach with just a single adaptive parameter. In order to manage wonders like nonlinear uncertainties, dynamic disturbances, and obscure time delays, class of single input single output strict-feedback nonlinear systems are introduced. Lyapunov– Krasovskii function approach is utilized to compensate the obscure time delays in the system. By combining the advances of the hyperbolic digression function with adaptive fuzzy backstepping technique, the controller guaranteed the semi-global boundedness of the considerable number of signals in the closed-circle system from the mean square point of view.

Jianbin et al. [16] have introduced H_∞ control for a class of nonlinear spatially distributed systems. It was described by first-order hyperbolic incomplete differential equations alongside Markovian bouncing actuator deficiencies. By utilizing T-S fuzzy models, the nonlinear hyperbolic PDE systems are right off the bat communicated with parameter uncertainties and dependable distributed fuzzy static output feedback controller are designed to guarantee the stochastic exponential strength of the resulting closed-circle system with certain H_∞ disturbance attenuation performance. In light of a Markovian Lyapunov functional combined with some framework disparity convexification techniques, a solid fuzzy static output feedback controller design are produced for the underlying fuzzy PDE systems. The controller picks up are acquired by explaining an arrangement of finite linear network disparities in light of the limited difference strategy in space.

Hongyi et al. [17] have actualized interim sort 2 Takagi–Sugeno (T– S) fuzzy model to represent uncertain nonlinear systems. By utilizing the lower and upper participation functions, the uncertain parameters were described. To break down the sliding motion, a basic sliding mode surface was designed. In light of the sliding mode surface, a sliding mode controller was designed to guarantee that the closed-loop system is uniformly limited. Simulation results were illustrated the effectiveness of the displayed control scheme.

III. T-S FUZZY FEEDBACK CONTROL SYSTEM

In this proposed system, Reliable Takagi-Sugeno fuzzy controller via state feedback and output feedback controller to provide robust stabilization with parameter uncertainty and disturbances. This system provides stabilization by the reliable consideration of disturbances and also considers the feedback from the output of controller thereby proving the reliability of the system. This T-S fuzzy model will bridges the gap between linear and non-linear control system. And the T-S fuzzy model is developed to provide robust control for the non-linear dynamic system. T-S Fuzzy model is developed with system parameters such as,

- Time varying uncertainty,
- External and internal disturbances,
- Delayed input and noise.

Then the reliable state feedback and output control feedback fuzzy controller is developed to stabilize and control the system for different uncertainties. The necessary conditions for stabilization are based on the quadratic affine Lyapunov function approach, which is less conservative than the common Lyapunov function. The stability functions are expressed in the form of Linear Matrix Inequalities. The stabilization of T-S fuzzy control is designed via state feedback and output feedback control, to guarantee the stabilization, control and analytical proof.

IV. PROBLEM FORMULATION

The T-S fuzzy model is an interposition of various linear and non-linear models through membership function. The fuzzy model is described by fuzzy If-Then rules and employed here to deal with the control design problem for the non-linear system. Consider a nonlinear system that can be represented by the following Takagi–Sugeno (T–S) fuzzy model.

Plant Rule i: IF $\phi_1(t)$ is P_1^i *AND* $\phi 2(t) = P_2{}^i$ …… *AND* $\phi_f(t)$ is P_f^i

THEN

$$\dot{x}(t) = (A_i + \Delta A_i(t))x(t) + (B_i + \Delta B_i(t))u(t) + D_i v(t)$$
$$y(t) = (C_i + \Delta C_i(t))x(t) \qquad i \in \{1,2,......w\} \quad (1)$$

Where, $\phi(t) = [\phi_1(t),..............\phi_f(t)]$ is the variable vector, $P_j^i,\ i = 1,.......w,\ \ j = 1,.......f$, are the fuzzy sets and u is the number of IF-THEN rules. $x(t) \in R^g$ is the state. $y(t) \in R^h$ is the controlled output, $u(t) \in R^r$ is the control input and $v(t) \in R^i$ is the external disturbance. $A_i, B_i, C_i andD_i$ are known real constant matrices and $\Delta A_i, \Delta B_i and\Delta C_i$ are real-valued unknown matrices representing time-varying parameter uncertainties.

In different practical models, either the state or the control input may be corrupted by the noise. In a practical model with the control input depend noise can be found which comes from the control system.

The firing interval of the i[th] rule is as given below,

$$\varpi(x(t)) = \left[\prod_{j=1}^{f} \underline{\mu}_{P_S^i}\left(P_j^i(x(t))\right), \prod_{j=1}^{f} \overline{\mu}_{P_S^i}\left(P_j^i(x(t))\right)\right] = \left[\underline{\varpi}_i(x(t)), \overline{\varpi}_i(x(t))\right] \quad (2)$$

where $\underline{\varpi}_i(x(t))$ indicates the lower grade of membership function and $\overline{\varpi}_i(x(t))$ indicates the upper grades of membership function, $\underline{\mu}_{P_s^i}\left(P_j^i(x(t))\right)$ stands for lower membership functions and $\overline{\mu}_{P_s^i}\left(P_j^i(x(t))\right)$ stands for upper membership functions. Now, $\overline{\mu}_{P_s^i}\left(P_j^i(x(t))\right) \geq \underline{\mu}_{P_s^i}\left(P_j^i(x(t))\right) \geq 0$ and $\overline{\varpi}_i(x(t)) \geq \underline{\varpi}_i(x(t)) \geq 0$ for all n. The TS fuzzy system can be written as,

$$\dot{x}(t) = \sum_{n=1}^{w} \varpi_i(x(t))\left[(A_i + \Delta A_i)x(t) + (B_i + \Delta B_i)u(t) + D_i v(t)\right] \quad (3)$$

$$y(t) = \sum_{n=1}^{w} \varpi_i(x(t))\left[C_i + \Delta C_i(x(t))\right] \quad (4)$$

Where, $\varpi_i(x(t)) = \underline{\tau}(x(t))\underline{\varpi}_i(x(t)) + \overline{\tau}_i(x(t))\overline{\varpi}_i(x(t)) \geq 0,\ \forall i$,

$$\sum_{i=1}^{w} \varpi_i(x(t)) = 1, \quad (5)$$

$$0 \leq \underline{\tau}_i(x(t)) \leq 1,\ \forall i, \quad (6)$$

$$0 \leq \overline{\tau}_i(x(t)) \leq 1,\ \forall i, \quad (7)$$

$$\underline{\tau}_i(x(t)) + \overline{\tau}_i(x(t)) = 1, \forall i \quad (8)$$

Where, $\underline{\tau}_i(x(t))$ and $\overline{\tau}_i(x(t))$ is nonlinear functions, and $\varpi_i(x(t))$ indicate the membership functions grades.

V. T-S FUZZY STATE FEEDBACK CONTROL

The T-S fuzzy system and T-S fuzzy state feedback controller doesn't share the same membership functions. The m^{th} rule of fuzzy controller is of the subsequent form:

Controller Rule *m*: IF $\Im_1(t)$ is S_1^m AND $\Im_2(t) = S_2{}^m$ *AND* $\Im_f(t)$ is S_f^m

THEN

$$u(t) = K * y(t) \quad (9)$$

Where, $\Im(t) = [\Im_1(t),..............\Im_f(t)]$ is the variable vector, $S_j^m,\ m = 1,.......w,\ \ j = 1,.......f$, are the fuzzy sets, $Z_m \in H^{p \times q}$ indicate the state feedback gain matrix of the rule m. The firing interval of m^{th} rule is given below,

$$\chi_m(x(t)) = \left[\prod_{j=1}^{f} \underline{\mu}_{S^m}\left(S_j^m(x(t))\right), \prod_{s=1}^{f} \overline{\mu}_{S^m}\left(S_j^m(x(t))\right)\right] = \left[\underline{\chi}_m(x(t)), \overline{\chi}_m(x(t))\right] \quad (10)$$

where $\underline{\chi}_m(x(t))$ indicates the lower grades of membership and $\overline{\chi}_m(x(t))$ indicates the upper grades of membership, $\underline{\mu}_{S^m}\left(S_j^m(x(t))\right)$ gives the lower membership functions and $\overline{\mu}_{S^m}\left(S_j^m(x(t))\right)$ gives the upper membership functions. At this time, $\overline{\mu}_{S^m}\left(S_j^m(x(t))\right) \geq \underline{\mu}_{S^m}\left(S_j^m(x(t))\right) \geq 0$ and $\overline{\chi}_m(x(t)) \geq \underline{\chi}_m(x(t)) \geq 0$ for all *j*. The TS fuzzy state feedback control statement can be written as,

$$u(t) = \sum_{m=1}^{w} \chi_m(x(t))K * y(t) \quad (11)$$

Where,

$$\chi_m(x(t)) = \frac{\underline{g}_m(x(t))\underline{\chi}_m(x(t)) + \overline{g}_m(x(t))\overline{\chi}_m(x(t))}{\sum_{k=1}^{w} \underline{g}_k(x(t))\underline{\chi}_k(x(t)) + \overline{g}_k(x(t))\overline{\chi}_k(x(t))} \geq 0,\ \forall m \quad (12)$$

$$\sum_{m=1}^{w} \chi_m(x(t)) = 1 \quad (13)$$

$$0 \leq \underline{g}_m(x(t)) \leq 1,\ \forall m \quad (14)$$

$$0 \leq \overline{g}_m(x(t)) \leq 1,\ \forall m \quad (15)$$

$$\underline{g}_m(x(t)) + \overline{g}_m(x(t)) = 1,\ \forall m \quad (16)$$

Where, $\underline{g}_m(x(t))$ and $\overline{g}_m(x(t))$ being predefined functions, and $\chi_m(x(t))$ gives the grades of membership of

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embedded membership functions. Subsequent representations for membership functions are,

$$\varpi_i\left(x(t)\right) \cong \varpi_i \qquad (17)$$

$$\chi_m\left(x(t)\right) \cong \chi_m \qquad (18)$$

Where, *i, m = 1,2,…w*. The resultant TS fuzzy closed loop system can be denoted as given by,

$$\dot{x}(t) = \sum_{i=1}^{w}\sum_{m=1}^{w} \varpi_i \chi_m \left[\left[A_i + \Delta A_i + \left(B_i + \Delta B_i\right)K * y(t)\right]x(t) + D_i v(t)\right] \qquad (19)$$

$$\dot{x}(t) = \sum_{i=1}^{w}\sum_{m=1}^{w} \varpi_i \chi_m \left[\left[A_i + \Delta A_i + \left(B_i + \Delta B_i\right)K * C_i\right]x(t) + D_i v(t)\right]$$

Where, $\sum_{i=1}^{w}\sum_{m=1}^{w} \varpi_i \chi_m = 1$.

$$u(t) = K * y(t)$$

VI. STABILITY ANALYSIS VIA QUADRATIC AFFINE LYAPUNOV APPROACH

Using the quadratic affine Lyapunov function, sufficient condition on stability analysis for the system in is presented for time varying uncertainty system. Then, an approach to the synthesis of the controller is given with matrix inequalities linearization procedures.

Statement 1:

The closed-loop T-S fuzzy systems with disturbances is globally asymptotically stable if there exists two common positive definite matrices $P > 0$ with the decay rate δ such that

$$S_k^T P + P S_k + \delta P < 0 \qquad (20)$$

$$\left(\frac{S_{ij} + S_{ji}}{2}\right)^T P + P\left(\frac{S_{ij} + S_{ji}}{2}\right) + \delta P \le 0, \qquad (21)$$

Where $i = 1,2,..u$ & $j = 1,2,..u$ and $i < j$

$S_{ij} = A_i + B_i K_j C_j$, S_k is the Affine parameter (22)

Proof:

This is LMI problem and the output matrix C_i comes between the output feedback gain and positive definite matrix. After multiplying the inequalities in equation (20) and (21) by P^{-1}, a new variable V is defined and $V = P^{-1}$. Let the matrix V be

$$V = \begin{bmatrix} V_{11(p*p)} & 0_{p(n-p)} \\ 0_{(n-p)p} & V_{22(n-p)(n-p)} \end{bmatrix} \qquad (23)$$

There exists a nonsingular transformation matrix F_i, such that

$$C_i F_i = \left[E_{(p*p)} \quad 0_{p(n-p)}\right] \qquad (24)$$

Where $F_i = \left[F_{1i} \quad F_{2i}\right]$. During this transformation, the TS fuzzy model can be represented for the i^{th} model is

Plant Rule: IF $\phi_1(t)$ is P_1^i *AND* $\phi_2(t)$ is …… *AND* $\phi_f(t)$ is P_f^i

THEN

$$\dot{\hat{x}}(t) = \hat{A}_i \hat{x}(t) + \hat{B}_i u(t) + D_i v(t)$$

$$\hat{y}(t) = \hat{C}_i \hat{x}(t) \qquad i \in \{1,2,......u\} \qquad (25)$$

Where $\hat{A}_i = F_i^{-1} A_i F_i$, $\hat{B}_i = F_i^{-1} B_i$ and $\hat{C}_i = C_i F_i$.

Based on the transformation, the quadratic affine Lyapunov stability with variable $V > 0$ and decay rate δ can be given as follows:

$\hat{A}_i V + \hat{B}_i Y_i \hat{C}_i + D + (S_k)^T + \delta V < 0$ where $i = 1,2,...u$ (26)

$$A_i V + A_j V + B_i Y_j C_j + B_j Y_i C_i + D + (S_k)^T + 2\delta V \qquad (27)$$

Where $i = 1,2,..u$ & $j = 1,2,..u$ and $i < j$

$$Y_i = K_i V_{11} \text{ and } Y_j = K_j V_{11} \qquad (28)$$

From the above equation (28), the output feedback gain vectors can be represented as

$$K_i = Y_i V_{11}^{-1} \qquad (29)$$

The output feedback gain attained as $K_i = Y_i V_{11}^{-1}$ for the closed loop system with K as stable. This completes the proof.

Statement 2:

Consider a closed-loop T-S fuzzy system in (1), if there exist a positive definite matrix $P > 0, W_i, E_{iii}, E_{iji}, E_{jii} = E_{iij}^T, E_{ijk} = E_{kji}^T, E_{ikj} = E_{jki}^T, E_{jik} = E_{kij}^T$, such that the LMI is satisfied, then the continuous time T-S fuzzy system is stable via T-S fuzzy model based state feedback controller system.

$PA_i^T + A_i P + W_i^T B_i^T + B_i W_i < E_{iii}$ where $i = 1,2,...u$ (30)

$$2PA_i^T + PA_j^T + (W_i + W_j)^T B_i^T + W_i^T B_j^T + 2A_i P + B_i(W_i + W_j) + A_j P + B_j W_i < E_{iij} + E_{iji} + E_{iij}^T \qquad (31)$$

Where $i = 1,2,..u$ & $j = 1,2,..u$ and $i \ne j$

$$2P(A_i + A_j + A_k)^T + (W_i + W_j)^T B_k^T + (W_i + W_k)^T B_j^T + (W_j + W_k)^T B_i^T +$$

$$2(A_i + A_j + A_k)P + B_k(W_i + W_j) + B_j(W_i + W_k) + B_i(W_j + W_k) < E_{ijk} + E_{ikj} + E_{jik} + E_{ijk}^T + E_{ikj}^T + E_{jik}^T \qquad (32)$$

Where $i = 1,2,..u-2,\ j = i+1,2,...,u-1$ and $k = j+1,2,...u$

$$\begin{bmatrix} E_{1i1} & E_{1i2} & \cdots & E_{1iu} \\ E_{2i1} & E_{2i2} & \cdots & E_{2iu} \\ \vdots & \vdots & \vdots & \vdots \\ E_{ui1} & E_{ui2} & & E_{uiu} \end{bmatrix} \le 0, i = 1,2,....u \qquad (33)$$

And the T-S fuzzy state feedback controller gain is $K_i = W_i P^{-1}$

Proof: Let $V = P^{-1}$, $K_i = W_i P^{-1}$. Multiply V for eq (30) and (31) , we have

$$A_i^T V + K_i^T B_i^T C_i^T V + V A_i + V C_i B_i K_i < V E_{iii} V \qquad (34)$$

Where $i = 1,2,..u$ & $j = 1,2,..u$ and $i \ne j$

$$2A_i^T V + A_j^T V + (K_i + K_j)^T B_i^T C_i^T V + K_i^T B_j^T C_j^T V + V C_i B_i (K_i + K_j) + 2V A_i + V A_j + V C_j B_j K_i$$

$$< VE_{iij}V + VE_{iji}V + VE_{iij}^{T}V \quad (35)$$

Where $i = 1,2,..u$ & $j = 1,2,..u$ and $i \neq j$

$$2(A_i + A_j + A_k)^T + (K_i + K_j)^T B_k^T C_k^T V + (K_i + K_k)^T B_j^T C_j^T V + (K_j + K_k)^T B_i^T C_i^T V +$$
$$2V((A_i + A_j + A_k) + VC_k B_k (K_i + K_j) + VC_j B_j (K_i + K_k) + VC_i B_i (K_j + K_k)$$
$$< VE_{ijk}V + VE_{ikj}V + VE_{jik}V + VE_{ijk}^T V + VE_{ikj}^T V + VE_{jik}^T V \quad (36)$$

Where $i = 1,2,..u-2,\ j = i+1,2,...,u-1$ and $k = j+1,2,...u$

$$\begin{bmatrix} VE_{1i1}V & VE_{1i2}V & \dots & VE_{1iu}V \\ VE_{2i1}V & VE_{2i1}V & \cdots & VE_{2iu}V \\ \vdots & \vdots & \ddots & \vdots \\ VE_{ui1}V & VE_{ui1}V & \cdots & VE_{uiu}V \end{bmatrix} < 0 \quad (37)$$

There exists $\infty > 0$, one thing should be consider here, ∞ will be negative if the motion of the pendulum is in opposite direction, then the above equation (37) will be

$$\begin{bmatrix} VE_{1i1}V & VE_{1i2}V & \dots & VE_{1iu}V \\ VE_{2i1}V & VE_{2i1}V & \cdots & VE_{2iu}V \\ \vdots & \vdots & \ddots & \vdots \\ VE_{ui1}V & VE_{ui1}V & \cdots & VE_{uiu}V \end{bmatrix} < -\infty I \quad (38)$$

For the closed-loop system, we define a Lyapunov function as the following

$$R(t) = \dot{x}^T(t)Vx(t) \quad (39)$$

For the given control gain K, using the stability theory for non-linear time-invariant system, the system is quadratic ally stable if there exists P>0 such that

$$\dot{R}(t) = \dot{x}^T(t)Vx(t) + x^T(t)(t)V\dot{x}(t) \quad (40)$$

$$= \sum_{i=1}^{u} \psi_i^3 x^T(t)((A_i^T + K_i^T B_i^T C_i^T) + D)V + V((A_i + C_i B_i K_i) + D)x(t)$$
$$+ \sum_{i=1}^{u}\sum_{\substack{j=1 \\ i\neq j}}^{u} \psi_i^2 \psi_j x^T(t)(((2A_i^T + A_j^T + K_i^T B_i^T C_i^T + K_i^T B_j^T C_j^T + K_j^T B_i^T C_i^T) + D)V +$$
$$V((2A_i + A_j + C_i B_i K_i + C_j B_j K_i + C_i B_i K_j) + D))x(t)$$
$$+ \sum_{i=1}^{u-2}\sum_{j=i+1}^{u-1}\sum_{k=j+1}^{u} \psi_i \psi_j \psi_k x^T(t)(((2(A_i + A_j + A_k)^T + (K_i + K_j)^T B_k^T C_k^T + (K_i + K_k)^T B_j^T C_j^T + (K_j + K_k)^T B_i^T C_i^T) + D)V$$

$$+V((2(A_i + A_j + A_k) + C_k B_k (K_i + K_j) + C_j B_j (K_i + K_k) + C_i B_i (K_j + K_k)) + D)x(t) \quad (41)$$

Substituting eq (34) to (37), we get

$$\dot{R}(t) \leq \sum_{i=1}^{r} \psi_i^3 x^T(t) E_{iii} x(t) + \sum_{i=1}^{u}\sum_{\substack{j=1 \\ i\neq j}}^{u} \psi_i^2 \psi_j x^T(t)(E_{iij} + E_{iji} + E_{iij}^T)x(t)$$

$$+ \sum_{i=1}^{u-2}\sum_{j=i+1}^{u-1}\sum_{k=j+1}^{u} \psi_i \psi_j \psi_k x^T(t)(E_{ijk} + E_{ikj} + E_{jik} + E_{ijk}^T + E_{ikj}^T + E_{jik}^T)x(t) \quad (42)$$

$$= \psi_1 \begin{bmatrix} \psi_1 x \\ \psi_2 x \\ \vdots \\ \psi_u x \end{bmatrix}^T \begin{bmatrix} E_{111} & E_{112} & \cdots & E_{11u} \\ E_{211} & E_{212} & \cdots & E_{21u} \\ \vdots & \vdots & \ddots & \vdots \\ E_{u11} & E_{u12} & \cdots & E_{u1u} \end{bmatrix} \begin{bmatrix} \psi_1 x \\ \psi_2 x \\ \vdots \\ \psi_u x \end{bmatrix} + \psi_2 \begin{bmatrix} \psi_1 x \\ \psi_2 x \\ \vdots \\ \psi_u x \end{bmatrix}^T \begin{bmatrix} E_{121} & E_{122} & \cdots & E_{12u} \\ E_{221} & E_{222} & \cdots & E_{22u} \\ \vdots & \vdots & \ddots & \vdots \\ E_{u21} & E_{u22} & \cdots & E_{u2u} \end{bmatrix} \begin{bmatrix} \psi_1 x \\ \psi_2 x \\ \vdots \\ \psi_u x \end{bmatrix}$$

$$+ \cdots + \psi_r \begin{bmatrix} \psi_1 x \\ \psi_2 x \\ \vdots \\ \psi_u x \end{bmatrix}^T \begin{bmatrix} E_{1u1} & E_{1u2} & \cdots & E_{1uu} \\ E_{2u1} & E_{2u2} & \cdots & E_{2uu} \\ \vdots & \vdots & \ddots & \vdots \\ E_{uu1} & E_{uu2} & \cdots & E_{uuu} \end{bmatrix} \begin{bmatrix} \psi_1 x \\ \psi_2 x \\ \vdots \\ \psi_u x \end{bmatrix} \quad (43)$$

$$= \begin{bmatrix} \psi_1 x \\ \psi_2 x \\ \vdots \\ \psi_u x \end{bmatrix}^T \left(\sum_{i=1}^{u} \psi_i \begin{bmatrix} E_{1i1} & E_{1i2} & \cdots & E_{1iu} \\ E_{2i1} & E_{2i2} & \cdots & E_{2iu} \\ \vdots & \vdots & \ddots & \vdots \\ E_{ui1} & E_{ui2} & \cdots & E_{uiu} \end{bmatrix} \right) \begin{bmatrix} \psi_1 x \\ \psi_2 x \\ \vdots \\ \psi_u x \end{bmatrix} \leq -\infty \sum_{i=1}^{u} \psi_i^3 x^T(t) \leq -\infty x^T(t)x(t) \quad (44)$$

This completes the proof of the statement 2.

VII. THEORETICAL ANALYSIS, EXAMPLE 1

Example 1: Consider the inverted pendulum 1,

The motion equation for the inverted pendulum are described below,

$$\dot{x}_1(t) = x_2(t)$$
$$x_2(t) = \frac{g\sin(x_1(t)) - \frac{amlx_2^2(t)\sin(2x,(t)}{2} - a\cos(x_1(t))u(t)}{\frac{4l}{3} - aml\cos^2(x_1(t))} \quad (45)$$

Where, $x_1(t)$ denotes the angle of the pendulum, $g = 9.8\frac{m}{s^2}$ is gravity constant, $m = 0.2$ is the mass of the pendulum, $M = 0.5$ is the mass of the cart, $2l = 0.6$ is the length of the pendulum and $u = 1$ is the force applied to the cart

$$a = \frac{1}{(m+M)} = 1.4285$$

Equation 45 can be written as,

$$\dot{x}_2(t) = \frac{1}{\frac{4l}{3} - aml\cos^2(x_1(t))} * (g\sin(x_1(t)) - \frac{amlx_2(t)\sin(2x_1(t))}{2} x_2(t) - a\cos(x_1(t))u(t)) \quad (46)$$

The threshold values of $z_1(t)$, $z_2(t)$, $z_3(t)$ and $z_4(t)$ are given by,

$$z_1(t) = \frac{1}{\frac{4l}{3} - aml\cos^2(x_1(t))} = \frac{1}{0.4 - (0.0857(\cos^2 88))},$$
$$z_2(t) = \sin(x_1(t)) = \sin(88),$$
$$z_3(t) = x_2(t)\sin(2x_1(t)) = (90\sin(176)),$$
$$z_4(t) = \cos(x_1(t)) = \cos(88)$$

Where $x_1(t) \in [-\frac{\pi}{2}, \frac{\pi}{2}]$ and $x_2(t) \in [-\gamma, \gamma]$. The system is uncontrollable when $x_1(t) = \pm\frac{\pi}{2}$. The fuzzy model is maintain the controllability, consider the $x_1(t) \in [-88^0, 88^0]$ equation 46 can be written as,

$$\dot{x}_2(t) = z_1(t)\{gz_2(t) - \frac{aml}{2} z_3(t)x_2(t) - az_4(t)u(t)\}$$

Replace them $z_1(t) - z_4(t)$ with TS fuzzy model representation is,

$$\max_{x_1(t)} z_1(t) = \frac{1}{\frac{4l}{3} - aml\alpha^2} = \rho_1$$

Where, $\alpha = \cos(88^0)$

$$\min_{x_1(t)} z_1(t) = \frac{1}{\frac{4l}{3} - aml\alpha^2} = \rho_2$$

$z_1(t)$ Can be written as,

$$z_1(t) = \sum_{i=1}^{2} F_i(z_1(t))\rho_i \quad (47)$$

Where,

$$F_1(z_1(t)) = \frac{z_1(t)-\rho_2}{\rho_1-\rho_2} = \left(\frac{\frac{1}{0.4-(0.0857(\cos^2 88))} - 3.1818}{3.1807-3.1818} \right), \quad (48)$$

$$F_2(z_2(t)) = \frac{\rho_1 - z_1(t)}{\rho_1-\rho_2} = \left(\frac{3.1807 - \frac{1}{0.4-(0.0857(\cos^2 88))}}{3.1807-3.1818} \right), \quad (49)$$

The membership function, $F_1(z_1(t))$ and $F_2(z_1(t))$ are obtained from property of $F_1(z_1(t)) + F_2(z_1(t)) = 1$

Find the sector $[d_1, d_2]$ that of two lines d_1x_1 and d_2x_1 where the slopes are $d_1 = 1$ and $d_2 = \frac{2}{\pi}$. To represent the $\sin(x_1(t))$ as follows,

$$z_2(t) = \sin(x_1(t)) = \sum_{i=1}^{2} \mu_i(z_2(t))d_i)x_i(t) \quad (50)$$

The property of membership functions $[\mu_1(z_2(t)) + \mu_2(z_2(t)) = 1]$, the membership function can be expressed as,

$$\mu_1(z_2(t)) = \begin{cases} \dfrac{z_2(t) - (\frac{2}{\pi})\sin^{-1}(z_2(t))}{(1-\frac{2}{\pi})\sin^{-1}(z_2(t))}, & z_2(t) \neq 0 \\ & otherwise \end{cases} \quad (51)$$

$$\mu_2(z_2(t)) = \begin{cases} \dfrac{\sin^{-1}(z_2(t)) - z_2(t)}{(1-\frac{2}{\pi})\sin^{-1}(z_2(t))}, & z_2(t) \neq 0 \\ & otherwise \end{cases} \quad (52)$$

Consider the $z_3(t) = x_2(t)\sin(2x,(t))$ since,

$$\max_{x_1(t),x_2(t)} z_3(t) = \gamma = e_1 \text{ and}$$

$$\max_{x_1(t),x_2(t)} z_3(t) = -\gamma = e_1$$

$z_1(t)$ can be derived in same way such as,

$$z_3(t) = x_2(t)\sin(2x,(t)) = \sum_{i=1}^{2} \upsilon_i(z(t))e_i \quad (53)$$

Where,

$$\upsilon_1(z_3(t)) = \frac{z_3(t)-e_2}{e_1-e_2} = \left(\frac{(90\sin(176)) - 65}{55-65} \right), \quad (54)$$

$$\upsilon_2(z_3(t)) = \frac{e_1 - z_3(t)}{e_1-e_2} = \left(\frac{55-(90\sin(176))}{55-65} \right), \quad (55)$$

$z_4(t)$ can derive in such a way

$$\max_{x_1(t)} z_4(t) = 1 = c_1 \quad \text{and} \quad \min_{x_1(t)} z_4(t) = \gamma = c_2,$$

$$z_4(t) = \cos(x_1(t)) = \sum_{i=1}^{2} P_i(x(t))c_i \quad (56)$$

Where

$$P_1(z_4(t)) = \frac{z_4(t)-c_2}{c_1-c_2} = \left(\frac{\cos(88)-\gamma}{1-\gamma} \right),$$

$$P_2(z_4(t)) = \frac{c_1 - z_4(t)}{c_1-c_2} = \left(\frac{1-\cos(88)}{1-\gamma} \right)$$

From equation (47-56) obtain the following TS fuzzy model for the inverted pendulum,

$$\begin{bmatrix} \dot{x}_1(t) \\ \dot{x}_2(t) \end{bmatrix} = \sum_{i=1}^{2}\sum_{j=1}^{2}\sum_{m=1}^{2}\sum_{n=1}^{2} F_i(z_1(t))\mu_j(z_2(t)\upsilon(z_3(t))P_n(z_4(t)) * \left(\begin{bmatrix} 0 & 1 \\ g.\rho_i d_j & \frac{-aml}{2}\rho_i d_m \end{bmatrix} \begin{bmatrix} x_1(t) \\ x_2(t) \end{bmatrix} + \begin{bmatrix} 0 \\ -a.\rho_i c_n \end{bmatrix} u(t) \right)$$

$$= \sum_{i=1}^{2}\sum_{j=1}^{2}\sum_{m=1}^{2}\sum_{n=1}^{2} F_i(z_1(t))\mu_j(z_2(t)\upsilon(z_3(t))P_n(z_4(t)) * \{A_{ijmn}x(t) + B_{ijmn}u(t) + D_{ijmn}\} \quad (57)$$

The equation (57) can be aggregated as one summation

$$\dot{x}(t) = \sum_{\Re=1}^{16} q_{\Re}(z(t))\{A'_{\Re}x(t) + B'_{\Re}u(t) + D'_{\Re}\} \quad (58)$$

$$\dot{x}(t) = \sum_{\Re=1}^{16} q_{\Re}(z(t))\{A'_{\Re}x(t) + B'_{\Re}(k * y(t)) + D'_{\Re}\}$$

$$y(t) = C'_{\Re}x(t)$$

$$\dot{x}(t) = \sum_{\Re=1}^{16} q_{\Re}(z(t))\{A'_{\Re}x(t) + B'_{\Re}(k * C'_{\Re}x(t)) + D'_{\Re}\}$$

$$\dot{x}(t) = \sum_{\Re=1}^{16} q_{\Re}(z(t))\{(A'_{\Re} + B'_{\Re})x(t) + D'_{\Re}\} \quad (59)$$

Where,

$$C = [b_1, b_2]$$

$$b_1 = 1,$$

$$b_2 = 0.0111$$

$$k = 10$$

$$D = \begin{bmatrix} 0.5 & 0.5 \\ 0.5 & 0.5 \end{bmatrix}$$

$$\Re = n + 2(m-1) + 4(j-1) + 8(i-1)$$

$$q_{\Re}(x(t)) = F_i(z_1(t)\mu_j(z_2(t)\upsilon_m(z_3(t))P_n(z_4(t))$$

$$A'_{\Re} = A_{ijmn},$$

$$B'_{\Re} = B_{ijmn},$$

In equation (58) indicate that the fuzzy model has the following rules.

Rule 1:

If $z_1(t)$ is "positive" and $z_2(t)$ is "zero" and $z_3(t)$ is "positive"' and $z_4(t)$ is "Big".

THEN

$$\dot{x}(t) = A'_1x(t) + B'_1u(t) + D'_1$$
$$y(t) = C'_1x(t)$$

Rule 2:

If $z_1(t)$ is "positive" and $z_2(t)$ is "zero" and $z_3(t)$ is "positive"' and $z_4(t)$ is "Small".

THEN

$$\dot{x}(t) = A'_2x(t) + B'_2u(t) + D'_2$$
$$y(t) = C'_2x(t)$$

Rule 3:

If $z_1(t)$ is "positive" and $z_2(t)$ is "zero" and $z_3(t)$ is "Negative"` and $z_4(t)$ is "Big".

THEN

$$\dot{x}(t) = A'_3 x(t) + B'_3 u(t) + D'_3$$
$$y(t) = C'_3 x(t)$$

Rule 4:

If $z_1(t)$ is "positive" and $z_2(t)$ is "zero" and $z_3(t)$ is "Negative"` and $z_4(t)$ is "Small".

THEN

$$\dot{x}(t) = A'_4 x(t) + B'_4 u(t) + D'_4$$
$$y(t) = C'_4 x(t)$$

Rule 5:

If $z_1(t)$ is "positive" and $z_2(t)$ is "Not zero" and $z_3(t)$ is "Positive"` and $z_4(t)$ is "Big".

THEN

$$\dot{x}(t) = A'_5 x(t) + B'_5 u(t) + D'_5$$
$$y(t) = C'_5 x(t)$$

Rule 6:

If $z_1(t)$ is "positive" and $z_2(t)$ is "Not zero" and $z_3(t)$ is "Positive"` and $z_4(t)$ is "Small".

THEN

$$\dot{x}(t) = A'_6 x(t) + B'_6 u(t) + D'_6$$
$$y(t) = C'_6 x(t)$$

Rule 7:

If $z_1(t)$ is "positive" and $z_2(t)$ is "Not zero" and $z_3(t)$ is "Negative"` and $z_4(t)$ is "Big".

THEN

$$\dot{x}(t) = A'_7 x(t) + B'_7 u(t) + D'_7$$
$$y(t) = C'_7 x(t)$$

Rule 8:

If $z_1(t)$ is "positive" and $z_2(t)$ is "Not zero" and $z_3(t)$ is "Negative"` and $z_4(t)$ is "Small".

THEN

$$\dot{x}(t) = A'_8 x(t) + B'_8 u(t) + D'_8$$
$$y(t) = C'_8 x(t)$$

Rule 9:

If $z_1(t)$ is "Negative" and $z_2(t)$ is "zero" and $z_3(t)$ is "positive" and $z_4(t)$ is "Big".

THEN

$$\dot{x}(t) = A'_9 x(t) + B'_9 u(t) + D'_9$$
$$y(t) = C'_9 x(t)$$

Rule 10:

If $z_1(t)$ is "Negative" and $z_2(t)$ is "zero" and $z_3(t)$ is "positive" and $z_4(t)$ is "Small".

THEN

$$\dot{x}(t) = A'_{10} x(t) + B'_{10} u(t) + D'_{10}$$
$$y(t) = C'_{10} x(t)$$

Rule 11:

If $z_1(t)$ is "Negative" and $z_2(t)$ is "zero" and $z_3(t)$ is "Negative" and $z_4(t)$ is "big".

THEN

$$\dot{x}(t) = A'_{11} x(t) + B'_{11} u(t) + D'_{11}$$
$$y(t) = C'_{11} x(t)$$

Rule 12:

If $z_1(t)$ is "Negative" and $z_2(t)$ is "zero" and $z_3(t)$ is "Negative" and $z_4(t)$ is "Small".

THEN

$$\dot{x}(t) = A'_{12} x(t) + B'_{12} u(t) + D'_{12}$$
$$y(t) = C'_{12} x(t)$$

Rule 13:

If $z_1(t)$ is "Negative" and $z_2(t)$ is "zero" and $z_3(t)$ is "Positive" and $z_4(t)$ is "Big".

THEN

$$\dot{x}(t) = A'_{13} x(t) + B'_{13} u(t) + D'_{13}$$
$$y(t) = C'_{13} x(t)$$

Rule 14:

If $z_1(t)$ is "Negative" and $z_2(t)$ is "Not zero" and $z_3(t)$ is "Positive" and $z_4(t)$ is "Small".

THEN

$$\dot{x}(t) = A'_{14} x(t) + B'_{14} u(t) + D'_{14}$$
$$y(t) = C'_{14} x(t)$$

Rule 15:

If $z_1(t)$ is "Negative" and $z_2(t)$ is "Not zero" and $z_3(t)$ is "Negative" and $z_4(t)$ is "Big".

THEN

$$\dot{x}(t) = A'_{15} x(t) + B'_{15} u(t) + D'_{15}$$
$$y(t) = C'_{15} x(t)$$

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Rule 16:
If $z_1(t)$ is "Negative" and $z_2(t)$ is "Not zero" and $z_3(t)$ is "Negative" and $z_4(t)$ is "Small".
Then:

$$\dot{x}(t) = A'_{16}x(t) + B'_{16}u(t) + D'_{16}$$
$$y(t) = C'_{16}x(t)$$

Where, $z_1(t), z_2(t), z_3(t)$ and z_4 are premise variables,

$$A'_1 = A_{1111} = \begin{bmatrix} 0 & 1.0000 \\ 31.1712 & -7.4974 \end{bmatrix},\; A'_2 = A_{1112} = \begin{bmatrix} 0 & 1.0000 \\ 31.1712 & -7.4974 \end{bmatrix}$$
$$A'_3 = A_{1121} = \begin{bmatrix} 0 & 1.0000 \\ 31.1712 & -8.8606 \end{bmatrix},\; A'_4 = A_{1122} = \begin{bmatrix} 0 & 1.0000 \\ 31.1712 & -8.8606 \end{bmatrix}$$
$$A'_5 = A_{1211} = \begin{bmatrix} 0 & 1.0000 \\ 0.3463 & -7.4974 \end{bmatrix},\; A'_6 = A_{1212} = \begin{bmatrix} 0 & 1.0000 \\ 0.3463 & -7.4974 \end{bmatrix}$$
$$A'_7 = A_{1221} = \begin{bmatrix} 0 & 1.0000 \\ 0.3463 & -8.8606 \end{bmatrix},\; A'_8 = A_{1222} = \begin{bmatrix} 0 & 1.0000 \\ 0.3463 & -8.8606 \end{bmatrix}$$
$$A'_9 = A_{2111} = \begin{bmatrix} 0 & 1.0000 \\ 31.1818 & -7.5000 \end{bmatrix},\; A'_{10} = A_{2112} = \begin{bmatrix} 0 & 1.0000 \\ 31.1818 & -7.5000 \end{bmatrix}$$
$$A'_{11} = A_{2121} = \begin{bmatrix} 0 & 1.0000 \\ 31.1818 & -8.8636 \end{bmatrix},\; A'_{12} = A_{2122} = \begin{bmatrix} 0 & 1.0000 \\ 31.1818 & -8.8636 \end{bmatrix}$$
$$A'_{13} = A_{2211} = \begin{bmatrix} 0 & 1.0000 \\ 0.3465 & -7.5000 \end{bmatrix},\; A'_{14} = A_{2212} = \begin{bmatrix} 0 & 1.0000 \\ 0.3465 & -7.5000 \end{bmatrix}$$
$$A'_{15} = A_{2221} = \begin{bmatrix} 0 & 1.0000 \\ 0.3465 & -8.8636 \end{bmatrix},\; A'_{16} = A_{2222} = \begin{bmatrix} 0 & 1.0000 \\ 0.3465 & -8.8636 \end{bmatrix}$$
$$B'_1 = B_{1111} = \begin{bmatrix} 0 \\ -4.5439 \end{bmatrix},\; B'_2 = B_{1112} = \begin{bmatrix} 0 \\ -4.5411 \end{bmatrix},\; B'_3 = B_{1121} = \begin{bmatrix} 0 \\ -4.5439 \end{bmatrix},$$
$$B'_4 = B_{1122} = \begin{bmatrix} 0 \\ -4.5411 \end{bmatrix},\; B'_5 = B_{1211} = \begin{bmatrix} 0 \\ -4.5439 \end{bmatrix},\; B'_6 = B_{1212} = \begin{bmatrix} 0 \\ -4.5411 \end{bmatrix}$$
$$B'_7 = B_{1221} = \begin{bmatrix} 0 \\ -4.5439 \end{bmatrix},\; B'_8 = B_{1222} = \begin{bmatrix} 0 \\ -4.5411 \end{bmatrix},\; B'_9 = B_{2111} = \begin{bmatrix} 0 \\ -4.5455 \end{bmatrix},$$
$$B'_{10} = B_{2112} = \begin{bmatrix} 0 \\ -4.5426 \end{bmatrix},\; B'_{11} = B_{2121} = \begin{bmatrix} 0 \\ -4.5455 \end{bmatrix},\; B'_{12} = B_{2122} = \begin{bmatrix} 0 \\ -4.5426 \end{bmatrix}$$
$$B'_{13} = B_{2211} = \begin{bmatrix} 0 \\ -4.5455 \end{bmatrix},\; B'_{14} = B_{2211} = \begin{bmatrix} 0 \\ -4.5426 \end{bmatrix},\; B'_{15} = B_{2221} = \begin{bmatrix} 0 \\ -4.5455 \end{bmatrix}$$
$$B'_{16} = B_{2222} = \begin{bmatrix} 0 \\ -4.5426 \end{bmatrix}$$

The membership function can be expressed below,

$$F_1(z_1(t)) = \frac{z_1(t) - \rho_2}{\rho_1 - \rho_2} = \left(\frac{\left(\frac{1}{0.4 - (0.0857)(\cos^2 88)} - 3.1818 \right)}{-0.0011} \right),$$
$$F_2(z_1(t)) = \frac{\rho_1 - z_1(t)}{\rho_1 - \rho_2} = \left(\frac{3.1807 - \left(\frac{1}{0.4 - (0.0857)(\cos^2 88)} \right)}{-0.0011} \right), \quad (60)$$
$$\mu_1(z_2(t)) = \frac{\sin(x_1(t)) - \left(\frac{2}{\pi}\right) z_2(t)}{\left(1 - \frac{2}{\pi}\right) z_2(t)} = \left(\frac{\sin(88)\left(1 - \left(\frac{2}{\pi}\right)\right)}{\left(1 - \frac{2}{\pi}\right)\sin(88)} \right) = 1, \quad \mu_2(z_2(t)) = \frac{x_1(t) - z_2(t)}{\left(1 - \frac{2}{\pi}\right) z_2(t)} = 0, \quad (61)$$

$$\upsilon_1(z_3(t)) = \frac{z_3(t) - e_2}{e_1 - e_2}, \quad \upsilon_2(z_3(t)) = \frac{e_1 - z_3(t)}{e_1 - e_2}$$
$$P_1(z_4(t)) = \frac{z_4(t) - c_2}{c_1 - c_2}, \quad P_2(z_4(t)) = \frac{c_1 - z_4(t)}{c_1 - c_2}$$

VIII. THEORETICAL ANALYSIS, EXAMPLE 2

Example 2: Consider the inverted pendulum 2:

In inverted pendulum 2, $x_1(t)$ denotes the angle of the pendulum, $g = 9.8\frac{m}{s^2}$ is gravity constant, $m = 1.2$ is the mass of the pendulum, $M = 0.5$ is the mass of the cart, $2l = 0.6$ is the length of the pendulum and $u = 1$ is the force applied to the cart

$$a = \frac{1}{(m+M)} = 0.5882$$

Equation 1 can be written as,

$$\dot{x}_2(t) = \frac{1}{\frac{4l}{3} - aml\cos^2(x_1(t))} \times \left(g\sin(x_1(t)) - \frac{aml x_2(t)\sin(2x_1(t))}{2} x_2(t) - a\cos(x_1(t))u(t)\right) \quad (62)$$

$$k_1(t) = \frac{1}{\frac{4l}{3} - aml\cos^2(x_1(t))} = \frac{1}{0.4 - \left(0.5142(\cos^2 88)\right)},$$
$$k_2(t) = \sin(x_1(t)) = \sin(88),$$
$$k_3(t) = x_2(t)\sin(2x_1(t)) = \left(90\sin(176)\right),$$
$$k_4(t) = \cos(x_1(t)) = \cos(88)$$

The equation (58) can be rewritten in inverted pendulum 2:

$$\dot{x}(t) = \sum_{\Re=1}^{16} q_{\Re}(z(t))\{(A'_{\Re} + B'_{\Re})x(t) + D'_{\Re}\} \quad (63)$$

Where,

$$C = [b_1, b_2]$$
$$b_1 = 1,$$
$$b_2 = 0.0111$$
$$k = 10$$
$$D = \begin{bmatrix} 0.5 & 0.5 \\ 0.5 & 0.5 \end{bmatrix}$$
$$\Re = n + 2(m-1) + 4(j-1) + 8(i-1)$$
$$q_{\Re}(x(t)) = F_i(z_1(t)\mu_j(z_2(t)\upsilon_m(z_3(t))P_n(z_4(t))$$
$$A'_{\Re} = A_{ijmn},$$
$$B'_{\Re} = B_{ijmn},$$

Equation (59) denotes the 16 rule fuzzy model which is described in inverted pendulum1.
The premise variables are,

$$A'_1 = A_{1111} = \begin{bmatrix} 0 & 1.0000 \\ 51.9892 & -30.8939 \end{bmatrix},\; A'_2 = A_{1112} = \begin{bmatrix} 0 & 1.0000 \\ 51.9892 & -30.8939 \end{bmatrix}$$

$$A'_3 = A_{1121} = \begin{bmatrix} 0 & 1.0000 \\ 51.9892 & -36.5110 \end{bmatrix}, A'_4 = A_{1122} = \begin{bmatrix} 0 & 1.0000 \\ 51.9892 & -36.5110 \end{bmatrix}$$

$$A'_5 = A_{1211} = \begin{bmatrix} 0 & 1.0000 \\ 0.5777 & -30.8939 \end{bmatrix}, A'_6 = A_{1212} = \begin{bmatrix} 0 & 1.0000 \\ 0.5777 & -30.8939 \end{bmatrix}$$

$$A'_7 = A_{1221} = \begin{bmatrix} 0 & 1.0000 \\ 0.5777 & -36.5110 \end{bmatrix}, A'_8 = A_{1222} = \begin{bmatrix} 0 & 1.0000 \\ 0.5777 & -36.5110 \end{bmatrix}$$

$$A'_9 = A_{2111} = \begin{bmatrix} 0 & 1.0000 \\ 52.0625 & -30.9375 \end{bmatrix}, A'_{10} = A_{2112} = \begin{bmatrix} 0 & 1.0000 \\ 52.0625 & -30.9375 \end{bmatrix}$$

$$A'_{11} = A_{2121} = \begin{bmatrix} 0 & 1.0000 \\ 52.0625 & -36.5625 \end{bmatrix}, A'_{12} = A_{2122} = \begin{bmatrix} 0 & 1.0000 \\ 52.0625 & -36.5625 \end{bmatrix}$$

$$A'_{13} = A_{2211} = \begin{bmatrix} 0 & 1.0000 \\ 0.5785 & -30.9375 \end{bmatrix}, A'_{14} = A_{2212} = \begin{bmatrix} 0 & 1.0000 \\ 0.5785 & -30.9375 \end{bmatrix}$$

$$A'_{15} = A_{2221} = \begin{bmatrix} 0 & 1.0000 \\ 0.5785 & -36.5625 \end{bmatrix}, A'_{16} = A_{2222} = \begin{bmatrix} 0 & 1.0000 \\ 0.5785 & -36.5625 \end{bmatrix}$$

$$B'_1 = B_{1111} = \begin{bmatrix} 0 \\ -3.1206 \end{bmatrix}, B'_2 = B_{1112} = \begin{bmatrix} 0 \\ -3.1186 \end{bmatrix}, B'_3 = B_{1121} = \begin{bmatrix} 0 \\ -3.1206 \end{bmatrix},$$

$$B'_4 = B_{1122} = \begin{bmatrix} 0 \\ -3.1186 \end{bmatrix}, B'_5 = B_{1211} = \begin{bmatrix} 0 \\ -3.1206 \end{bmatrix}, B'_6 = B_{1212} = \begin{bmatrix} 0 \\ -3.1186 \end{bmatrix}$$

$$B'_7 = B_{1221} = \begin{bmatrix} 0 \\ -3.1206 \end{bmatrix}, B'_8 = B_{1222} = \begin{bmatrix} 0 \\ -3.1186 \end{bmatrix}, B'_9 = B_{2111} = \begin{bmatrix} 0 \\ -3.1250 \end{bmatrix},$$

$$B'_{10} = B_{2112} = \begin{bmatrix} 0 \\ -3.1230 \end{bmatrix}, B'_{11} = B_{2121} = \begin{bmatrix} 0 \\ -3.1250 \end{bmatrix}, B'_{12} = B_{2122} = \begin{bmatrix} 0 \\ -3.1230 \end{bmatrix}$$

$$B'_{13} = B_{2211} = \begin{bmatrix} 0 \\ -3.1250 \end{bmatrix}, B'_{14} = B_{2211} = \begin{bmatrix} 0 \\ -3.1230 \end{bmatrix}, B'_{15} = B_{2221} = \begin{bmatrix} 0 \\ -3.1250 \end{bmatrix}$$

$$B'_{16} = B_{2222} = \begin{bmatrix} 0 \\ -3.1230 \end{bmatrix}$$

The membership function can be described by,

$$F_1(z_1(t)) = \frac{z_1(t) - \rho_2}{\rho_1 - \rho_2} = \left(\frac{\left(\frac{1}{0.4 - (0.0857)(\cos^2 88)} - 5.3125 \right)}{-0.0075} \right),$$

$$F_2(z_1(t)) = \frac{\rho_1 - z_1(t)}{\rho_1 - \rho_2} = \left(\frac{5.3050 - \left(\frac{1}{0.4 - (0.0857)(\cos^2 88)} \right)}{-0.0075} \right), \tag{64}$$

$$\mu_1(z_2(t)) = \frac{\sin(x_1(t) - \left(\frac{2}{\pi}\right) z_2(t)}{\left(1 - \frac{2}{\pi}\right) z_2(t)} = \left(\frac{\sin(88)\left(1 - \left(\frac{2}{\pi}\right)\right)}{\left(1 - \frac{2}{\pi}\right)\sin(88)} \right) = 1$$

$$\mu_2(z_2(t)) = \frac{x_1(t) - z_2(t)}{\left(1 - \frac{2}{\pi}\right) z_2(t)} = 0$$

IV. EXPERIMENTAL RESULTS

In this section, two examples are provided to illustrate the Fuzzy output feedback design approach for TS fuzzy systems, two inverted pendulum examples are given in this section which is implemented on Matlab working platform. We have taken two inverted pendulum that represents better a human-like gait. It can be widely applied for the bipedal gait control and balance recovery. For that purpose, we considering that for demonstrating the Fuzzy output feedback design approach for TS fuzzy systems.

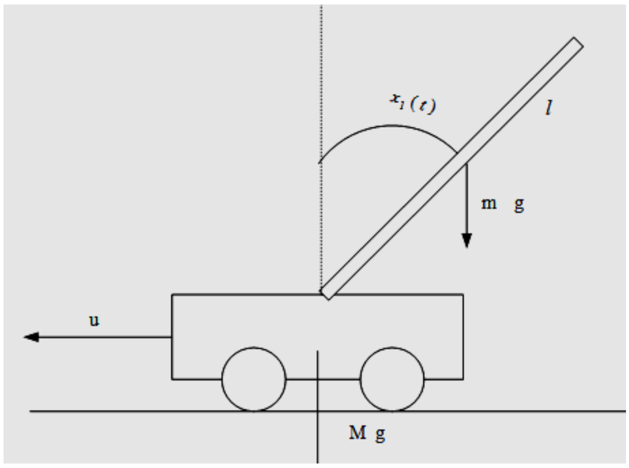

Figure 1: Inverted Pendulum model

Prior to applying the sector nonlinearity approach, it is often a good practice to simplify the original nonlinear model as much as possible. This procedure is important for practical applications because it always leads to the reduction of the number of model rules, which reduces the effort for analysis and design of fuzzy control systems.

The experimental results in figure 2 to 6 are obtained by applying our proposed inverted pendulum 1 values.

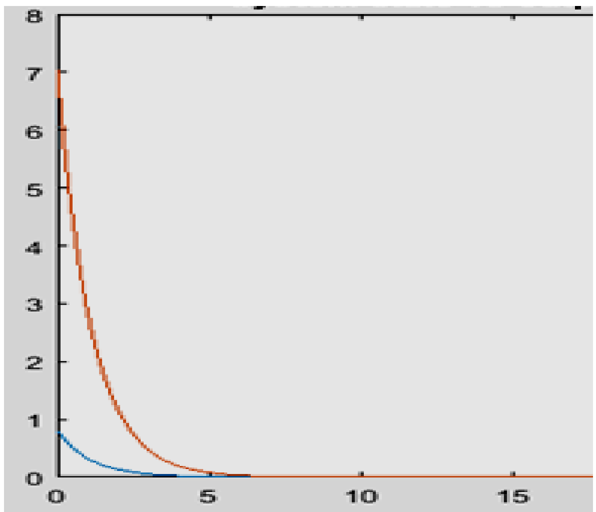

Figure 2: System state Vs Output state (Pendulum 1)

The system state and output state are presented in Figure 2. It can be perceived that the system outputs meet to a small neighborhood around zero. To verify the legitimacy of the designed observer, the system states and observer states are described in Figure 2. It can be seen that the designed system state is very good for approximating the unmeasured state.

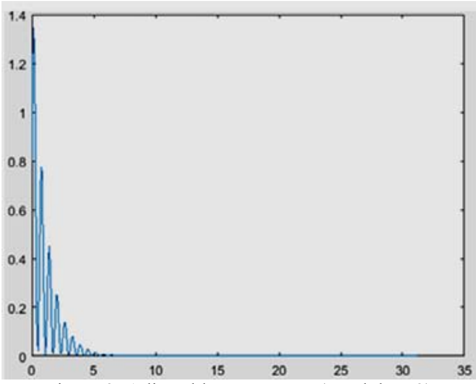
Figure 3: Adjustable Parameters (Pendulum 2)

In Figure 3 that the tracking performance is good and the anticipated and actual parameters are nearly identical. It should be stated that it has both stability and better performance is attained when using the proposed TS fuzzy system.

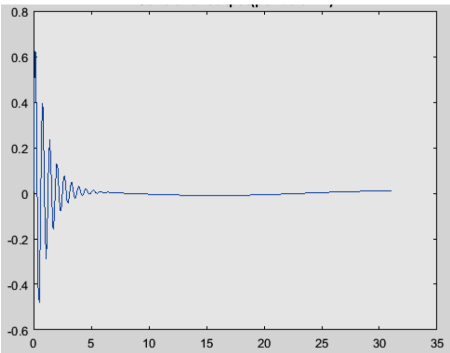
Figure 4: Controller output (Pendulum 1)

Boundedness of the controllers u_1 and u_2 is illustrated by Figure 4. The controller output also have limitation.

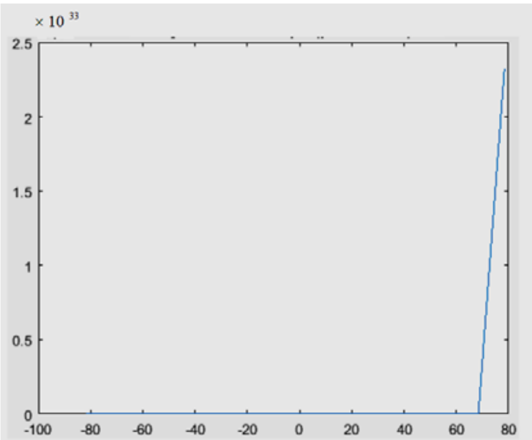
Figure 5: Fuzzy controller output (Pendulum 1)

The designed fuzzy controller shares the same fuzzy sets with the fuzzy model in the premise parts. The fuzzy controller design is to determine the local feedback gains in the consequent parts.

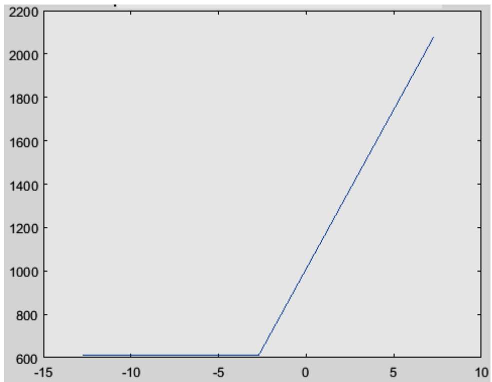
Figure 6: Mapping input and output function (Pendulum 1)

The experimental results in figure 7 to 11 are obtained by applying our proposed inverted pendulum 2 values.

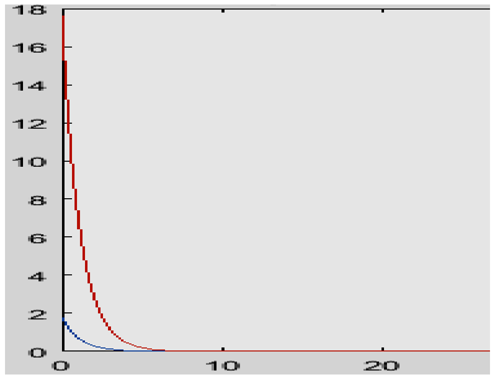
Figure 7: System state Vs output state (Pendulum 2)

To verify the legitimacy of the designed observer, the system states and observer states are described in Figure 7. It can be seen that the designed system state is very good for approximating the unmeasured state.

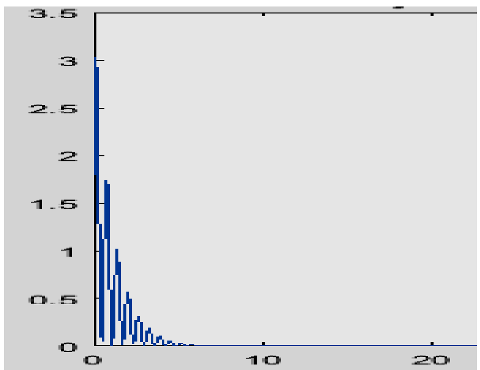
Figure 8: Adjustable parameters (Pendulum 2)

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In Figure 8 that the tracking performance is good and the anticipated and actual parameters are nearly identical. It should be stated that it has both stability and better performance is attained when using the proposed TS fuzzy system.

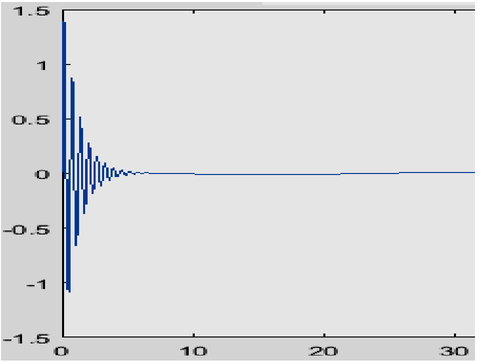

Figure 9: Controller output (Pendulum 2)

Boundedness of the controllers u_1 and u_2 is illustrated by Figure 9. The controller output also have limitation.

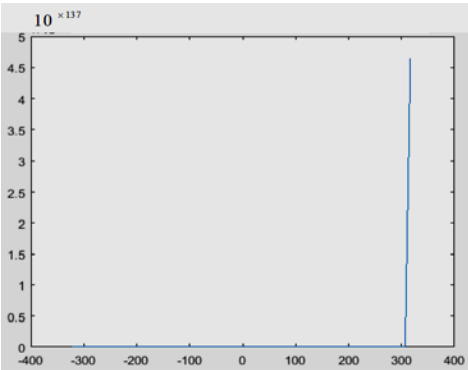

Figure 10: Fuzzy controller output (Pendulum 2)

In Figure 10 describes the fuzzy controller output. The designed fuzzy controller shares the same fuzzy sets with the fuzzy model in the premise parts. The fuzzy controller design is to determine the local feedback gains in the consequent parts.

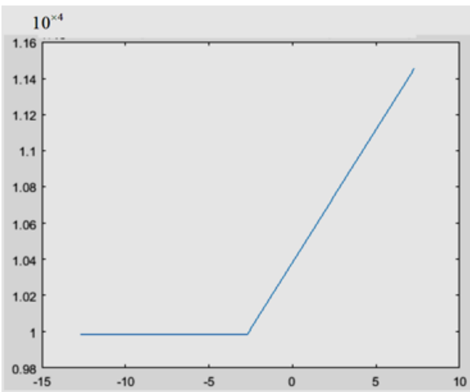

Figure 11: Mapping input and output function (Pendulum 2)

Overall, these results obtained from the simulations shows that our proposed TS fuzzy control design are very efficient to control the uncertain nonlinear systems with disturbances and parameter uncertainties.

A. Comparison of Results

The results of the proposed method and existing controller in ref [16] are compared in Table 1. This shows that the proposed controller is better than [16] for providing stability to the system for varying time uncertainties subjected to bounded external disturbance.

TABLE 1: COMPARISON OF RESULTS

Parameters	Fuzzy-Model-Based Reliable Static Output Feedback H∞ Control	Proposed Method	
		Pendulum 1	Pendulum 2
Control gain (msec)	8	4	5

From the above table we attain the better stabilization through our proposed method compared to the conventional controller. The Fuzzy-Model-Based Reliable Static Output Feedback H∞ Control has taken 8 msec to attain the better convergence for control gain. Our proposed controller have taken 4 and 5 msec for pendulum 1 and pendulum 2 respectively. They obtain the fast convergence for the stabilization compared to the previous method.

X. CONCLUSION

In this proposed method, we have design a Takagi-Sugeno fuzzy systems with parameter uncertainties and disturbances via state feedback under the conditions that the state variables are unavailable for measurement. The necessary conditions for stabilization are expressed in the form of Linear Matrix Inequalities based quadratic affine Lyapunov function approach, which is less conservative than the common Lyapunov function. For any external disturbances fuzzy states while keeping a suitably small number of fuzzy rules. The proposed method is implemented on Matlab and the result section shows that the proposed TS fuzzy controller is efficiently controlled the uncertain nonlinear system. Also, in order to prove the effectiveness of the proposed TS fuzzy controller, we applied to a practical inverted pendulum 1 and 2 and analyzed the results.

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