Computer Experiments of Digital IIR Filter Design Procedure using Chebyshev-I Approach and Impulse Invariance Procedure

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***Abstract* - The digital IIR (Infinite Impulse Response) filter has been one of the crucial elements in telecommunication and Digital Signal Processing (DSP) and design procedures for it have been proposed in terms of precision, computation time and implementation complexity. In this paper we undetake detailed analyses through computer experiments of the IIR filter design procedure using Chebyshev-I approach with impulse invariance procedure. In the algebraic computer simulation, two investigated experiments of low pass IIR filter design procedures are analyzed in both algebraic and computer processing. From the Chebyshev-I procedure, the magnitude, in decibels dB, and the phase of the frequency response between the digital filter and the analog filter are analyzed in depth to demonstrate their implementation characteristics.**

Keywords - digital IIR (Infinite Impulse Response) filter, Chebyshev Type I - Filter Desiring Technique, Impulse Invariance Concept, Digital Signal Processing (DSP).

I. INTRODUCTION

Traditionally, the expression spectrum-selective filter delineates a system [1,6,7,10] that is one of the extensive set of LTI (Linear Time-Invariant) systems, which permits fractional spectrum and ultimately stops all other spectrum. However, the broader definition of filter [2,3,8,9,11] is the system, which manipulates fractional spectrum [4,5] but ultimately permits all other spectrum. Because of its frequency respond appearance, the digital IIR (Infinite Impulse Response) filter [2,3] is one of the crucial telecommunication elements [12,13,16] hence there are a lot of designing procedures [2,3,15] of digital IIR filter that been wildly investigated during that three and half decades from technical remaining point of view for instant precision, calculated time, implemented complexity [17] or etc. From algebraic point of view [4,5], one of the simplest and most ubiquitous filter desiring procedure is the Chebyshev-I idea [2,3,12,13,14,16], has been implemented for building analog filter system function. Next, the digital filter system function is algebraically defined from the analog filter system function by the impulsive invariant procedure [15]. Hence, this article ultimately analyses the computer experiment of the digital IIR filter design procedure placed on Chebyshev-I approach and impulse invariance procedure.

II. MATHEMATICAL THEORY OF CHEBYSHEV-I APPROACH

The property of the Chebyshev-I low-pass filter is that the magnitude response is flat characteristic in stopband but equiripple characteristic in passband. The magnitude squared response ($\left|H_c(j\Omega)\right|^2$) of this Chebyshev-I low-pass filter can be mathematically expressed as:

$$\left|H_c(j\Omega)\right|^2 = \frac{1}{1+\varepsilon^2 T_N^2\left(\frac{\Omega}{\Omega_c}\right)} \tag{1}$$

where N is the order of the Chebyshev-I low-pass filter and Ω_c is the cutoff frequency (rad/sec). $T_N(\cdot)$ is the function of the N^{th} order Chebyshev polynomial, which can be mathematically expressed as:

$$T_N\left(\frac{\Omega}{\Omega_c}\right) = \begin{cases} \cos\left(N\cos^{-1}\left(\frac{\Omega}{\Omega_c}\right)\right) & ,0 \le \left(\frac{\Omega}{\Omega_c}\right) \le 1 \\ \cosh\left(N\cosh^{-1}\left(\frac{\Omega}{\Omega_c}\right)\right) & ,1 \le \left(\frac{\Omega}{\Omega_c}\right) \le +\infty \end{cases} \tag{2}$$

For $0 \le \left(\frac{\Omega}{\Omega_c}\right) \le 1$, $T_N\left(\frac{\Omega}{\Omega_c}\right) = \cos\left(N\cos^{-1}\left(\frac{\Omega}{\Omega_c}\right)\right)$

therefore this equation can be expressed in the recursive form by using the trigonometric identities as:

$$T_{N+1}\left(\frac{\Omega}{\Omega_c}\right) = \left(2\left(\frac{\Omega}{\Omega_c}\right)\right)\times T_N\left(\frac{\Omega}{\Omega_c}\right) - T_{N-1}\left(\frac{\Omega}{\Omega_c}\right) \tag{3}$$

The design of the Chebyshev-I low-pass filter can be expressed as following steps:

1. Determine the order of Chebyshev-I low-pass filter, N, from the specification: R_p (passband ripple parameter) and A_s (stopband attenuation parameter):

$$\varepsilon = \sqrt{10^{\frac{-R_p}{10}} - 1} \tag{4}$$

$$A = 10^{\frac{-A_s}{20}} \tag{5}$$

$$g = \sqrt{\frac{\left(A^2 - 1\right)}{\varepsilon^2}} \tag{6}$$

$$\Omega_r = \frac{\Omega_s}{\Omega_p} \tag{7}$$

$$N = \left\lceil \frac{\log_{10}\left(g + \sqrt{g^2 - 1}\right)}{\log_{10}\left(\Omega_r + \sqrt{\Omega_r^2 - 1}\right)} \right\rceil \tag{8}$$

Where:

- $\lceil \cdot \rceil$ is the round up operator.
- R_p is the passband ripple parameter (dB) or $R_p = -10\log\left(\frac{1}{1+\varepsilon^2}\right)$ when ε is the passband ripple parameter ($\frac{1}{1+\varepsilon^2} \le \left|H_c(j\Omega)\right|^2 \le 1, \quad |\Omega| \le \Omega_p$).
- A_s is the stopband attenuation parameter (dB) or $A_s = -10\log\left(\frac{1}{A^2}\right)$ when A the stopband attenuation parameter ($0 \le \left|H_c(j\Omega)\right|^2 \le \frac{1}{A^2}, \quad \Omega_s \le |\Omega| \le \Omega_p$).

2. Determine the filter parameter Ω_c (or the cutoff frequency of the Chebyshev-I CT filter) from the specification: Ω_p.

$$\Omega_c = \Omega_p \tag{9}$$

3. Determine the poles of the system function of the Chebyshev-I CT filter (from the filter parameter ε, N and Ω_c)

$$\alpha = \frac{1}{\varepsilon} + \sqrt{1 + \frac{1}{\varepsilon^2}} \tag{10}$$

$$a = 0.5\left(\alpha^{1/N} - \alpha^{-1/N}\right) \tag{11}$$

$$b = 0.5\left(\alpha^{1/N} + \alpha^{-1/N}\right) \tag{12}$$

Therefore, the poles (p_k) of this system function with stable property (The stable filter $H_c(s)$ can be defined by limiting poles in the left half-plain.) can be mathematically expressed as:

$$p_k = \left(\begin{array}{l} (a\Omega_c)\cos\left(\frac{\pi}{2} + \frac{(2k+1)\pi}{2N}\right) \\ +j(b\Omega_c)\sin\left(\frac{\pi}{2} + \frac{(2k+1)\pi}{2N}\right) \end{array} \right), k = 0,1,2,\ldots,(N-1) \tag{13}$$

The example of the pole plot of the system function of the Chebyshev-I CT filter is illustrated in the following figure for $N = 3$ and $N = 4$.

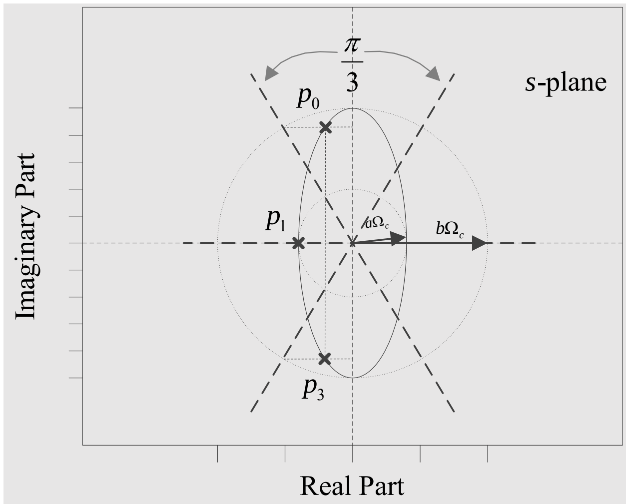

Figure 1. The pole plot of the system function of the Chebyshev-I CT filter for $N = 3$.

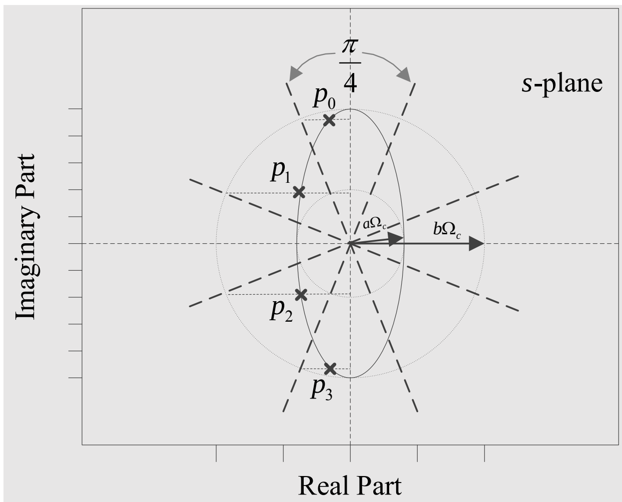

Figure 2. The pole plot of the system function of the Chebyshev-I CT filter for $N = 4$.

4. Determine the system function ($H_c(s)$) of the Chebyshev-I CT filter (from the estimated poles):

$$H_c(s) = \frac{K\prod_{\text{LHP}} p_k}{\prod_{\text{LHP}} (s - p_k)} \tag{14}$$

$$\text{where } K = \begin{cases} 1 & ,N \text{ odd} \\ \frac{1}{\sqrt{1+\varepsilon^2}} & ,N \text{ even} \end{cases} \tag{15}$$

III. MATHEMATICAL THEORY OF IMPULSE INVARIANCE APPROACH

From impulse invariance idea, the DT system impulse response ($h[n]$) is algebraically defined from the CT system impulse response ($h_c(t)$) by using the sampling method hence the DT system impulse response ($h[n]$) can be algebraically composed as:

$h[n]=T_d h_c(nT_d)$ where T_d is a period of sampling. (6)

From algebraic analysis both the CT Fourier analysis and the DT Fourier analysis, the CT system function can be algebraically composed as:

$$H\left(e^{j\omega}\right)=\sum_{k=-\infty}^{+\infty} H_c\left(j\frac{\omega}{T_d}+j\frac{2\pi}{T_d}k\right) \tag{7}$$

For the CT filter is bandlimited (or $H_c(j\Omega)=0 \quad ,|\Omega|\geq \pi/T_d$), the DT system function can be algebraically composed as:

$$H\left(e^{j\omega}\right)=\sum_{k=-\infty}^{+\infty} H_c\left(j\frac{\omega}{T_d}\right) \quad ,|\omega|\leq\pi \tag{8}$$

Hence, the main idea of the impulse invariance idea is for maintaining the aspect of the CT system impulse response ($h_c(t)$) by employing the sampling method hence the CT system is bandlimited (or $H_c(j\Omega)=0 \quad ,|\Omega|\geq \pi/T_d$) from that fact that the aliasing situation appears when the CT system is not bandlimited.

The algebraic projection from continuous domain plain and digital domain, which can be shown as following figure, can be algebraically written as:

- The $\mathrm{Re}(s)<0$ is algebraically projected into $\mathrm{Re}(z)<1$ (or inside the unit circle).
- The $\mathrm{Re}(s)=0$ is algebraically projected into $\mathrm{Re}(z)=1$ (at the boundary of the unit circle).
- The $\mathrm{Re}(s)>0$ is algebraically projected into $\mathrm{Re}(z)>1$ (or outside the unit circle).

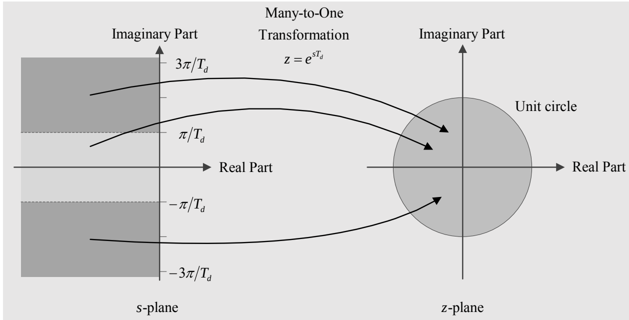

Figure 3. The impulse invariance mapping of the complex plane from the s-plane to z-plane.

The design of the DT IIR low-pass filter by using the impulse invariance can be expressed as following steps:

1. Determine the continuous frequency of passband (Ω_p) and stopband (Ω_s) from the specification: ω_p, ω_s and T_d

$$\Omega_p=\frac{\omega_p}{T_d} \quad (9) \qquad \text{and} \qquad \Omega_s=\frac{\omega_s}{T_d} \quad (10)$$

2. Determine the system function $H_c(s)$ of continuous-time low-pass filter (Butterworth filter, Chebyshev filter or Elliptic filter) from the specification: Ω_p, Ω_s, R_p and A_s (the detail of the continuous-time filter design is expressed in the preceding section).

3. Determine the system function $H_c(s)$ of continuous-time low-pass filter in the partial fraction expansion form, which can be mathematically expressed as:

$$H_c(s)=\sum_{k=1}^{N}\frac{R_k}{(s-p_k)} \tag{11}$$

4. Determine the system function $H(z)$ of discrete-time low-pass IIR filter from the system function $H_c(s)$ of continuous-time low-pass filter by using impulse invariance transformation for converting the continuous-time poles $\{p_k\}$ to be discrete-time low-pass poles $\{e^{p_k T_d}\}$. Consequently, the system function $H(z)$ of discrete-time low-pass IIR filter can be mathematically expressed as following

$$H(z)=\sum_{k=1}^{N}\frac{R_k}{\left(1-e^{p_k T_d}z^{-1}\right)} \tag{12}$$

The next section presents numerous examples of the designing of CT Butterworth low-pass filter by using the mathematical analysis for demonstrating the system function, the magnitude response and the phase response for examining the performance of this filter.

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IV. EXPERIMENTAL SIMULATION

All simulation outcomes are computed by the MATLAB software, which are operated by PC with CPU: Intel i7-6700HQ and RAM Memory: 16 GB.

A. Experimental Simulation Results for Case 1

By using the impulse invariance techniques, design the Chebyshev-I IIR filter where the passband gain ($0 \le |\omega| \le 0.2\pi$) between 0 dB and -7 dB, and stopband ($0.3\pi \le |\omega| \le \pi$) has attenuation of -16 dB where $T_d = 1$. Sketch the magnitude in decibels (dB), the magnitude and the phase of this frequency response of this Chebyshev-I IIR filter.
The design of the DT IIR low-pass filter by using the impulse invariance can be expressed as following step.

Step 1: Determine the continuous frequency of passband (Ω_p) and stopband (Ω_s) from the specification: ω_p, ω_s and T_d

$$\Omega_p = \frac{\omega_p}{T_d} \quad (9) \qquad \rightarrow \qquad \Omega_p = \frac{0.2\pi}{1} = 0.2\pi \qquad (16.1)$$

and

$$\Omega_s = \frac{\omega_s}{T_d} \quad (10) \qquad \rightarrow \qquad \Omega_s = \frac{0.3\pi}{1} = 0.3\pi \qquad (16.2)$$

Step 2: Determine the system function $H_c(s)$ of continuous-time lowpass filter (Chebyshev-I filter) from the specification: Ω_p, Ω_s, R_p and A_s (the detail of the continuous-time filter design is expressed in the preceding section)

Step 2.1 : Determine the order of Chebyshev-I lowpass filter, N, from the specification: R_p (passband ripple parameter) and A_s (stopband attenuation parameter)

$$\varepsilon = \sqrt{10^{\frac{-R_p}{10}} - 1} \quad (4) \qquad \rightarrow \qquad \varepsilon = \sqrt{10^{\frac{-(-7)}{10}} - 1}$$

$$\rightarrow \qquad \varepsilon = 2.0030$$

$$A = 10^{\frac{-A_s}{20}} \quad (5) \qquad \rightarrow \qquad A = 10^{\frac{16}{20}}$$

$$\rightarrow \qquad A = 6.3096$$

$$g = \sqrt{\frac{\left(A^2 - 1\right)}{\varepsilon^2}} \quad (6) \qquad \rightarrow \qquad g = \sqrt{\frac{\left(6.3096^2 - 1\right)}{2.0030^2}}$$

$$\rightarrow \qquad g = 3.1103$$

$$\Omega_r = \frac{\Omega_s}{\Omega_p} \quad (7) \qquad \rightarrow \qquad \Omega_r = \frac{3\pi}{2\pi}$$

$$\rightarrow \qquad \Omega_r = 1.5$$

From g and Ω_r, the order of Chebyshev-I low-pass filter, N, can be mathematically expressed as following.

$$N = \left\lceil \frac{\log_{10}\left(g + \sqrt{g^2 - 1}\right)}{\log_{10}\left(\Omega_r + \sqrt{\Omega_r^2 - 1}\right)} \right\rceil \qquad (8)$$

$$\rightarrow \qquad N = \left\lceil \frac{\log_{10}\left(3.1103 + \sqrt{3.1103^2 - 1}\right)}{\log_{10}\left(1.5 + \sqrt{1.5^2 - 1}\right)} \right\rceil$$

$$\rightarrow \qquad N = \lceil 1.8713 \rceil$$

$$\rightarrow \qquad N = 2 \qquad (16.3)$$

Step 2.2 : Determine the filter parameter Ω_c (or the cutoff frequency of the Chebyshev-I CT filter) from the specification: Ω_p.

$$\Omega_c = \Omega_p \quad (9) \qquad \rightarrow \qquad \Omega_c = 2\pi = 0.6283 \quad (16.4)$$

Step 2.3 : Determine the poles of the system function of the Chebyshev-I CT filter (from the filter parameter ε, N and Ω_c)

$$\alpha = \frac{1}{\varepsilon} + \sqrt{1 + \frac{1}{\varepsilon^2}} \quad (10) \qquad \rightarrow \alpha = \frac{1}{2.0030} + \sqrt{1 + \frac{1}{2.0030^2}}$$

$$\rightarrow \quad \alpha = 1.6170$$

$$a = 0.5\left(\alpha^{1/N} - \alpha^{-1/N}\right) \qquad (11)$$

$$\rightarrow \qquad a = 0.5\left((1.6170)^{1/2} - (1.6170)^{-1/2}\right)$$

$$\rightarrow \qquad a = 0.2426$$

$$b = 0.5\left(\alpha^{1/N} + \alpha^{-1/N}\right) \qquad (12)$$

$$\rightarrow \qquad b = 0.5\left((1.6170)^{1/2} + (1.6170)^{-1/2}\right)$$

$$\rightarrow \qquad b = 1.0290$$

Therefore, the poles (p_k) of this system function with stable property (The stable filter $H_c(s)$ can be defined by limiting poles in the left half-plain.) can be mathematically expressed as following.

$$p_k = \begin{pmatrix} (a\Omega_c)\cos\left(\frac{\pi}{2} + \frac{(2k+1)\pi}{2N}\right) \\ + j(b\Omega_c)\sin\left(\frac{\pi}{2} + \frac{(2k+1)\pi}{2N}\right) \end{pmatrix} \quad , k = 0,1,2,\ldots,(N-1) \qquad (13)$$

$$\rightarrow \qquad p_k = \begin{pmatrix} (0.2426 \times 0.6283)\cos\left(\frac{\pi}{2} + \frac{(2k+1)\pi}{2(2)}\right) \\ + j(1.0290 \times 0.6283)\sin\left(\frac{\pi}{2} + \frac{(2k+1)\pi}{2(2)}\right) \end{pmatrix} \quad , k = 0,1$$

$$\rightarrow \quad p_k = \left\{ \begin{array}{l} p_0 = \left(\begin{array}{l} (0.2426 \times 0.6283)\cos\left(\frac{\pi}{2} + \frac{\pi}{4}\right) \\ +j(1.0290 \times 0.6283)\sin\left(\frac{\pi}{2} + \frac{\pi}{4}\right) \end{array} \right) \\ , p_1 = \left(\begin{array}{l} (0.2426 \times 0.6283)\cos\left(\frac{\pi}{2} + \frac{3\pi}{4}\right) \\ +j(1.0290 \times 0.6283)\sin\left(\frac{\pi}{2} + \frac{3\pi}{4}\right) \end{array} \right) \end{array} \right\}$$

$$\rightarrow \quad p_k = \{ p_0 = (-0.1078 + j0.4572), p_1 = (-0.1078 - j0.4572) \} \quad (16.5)$$

The pole plot of the system function of the Chebyshev-I CT filter is illustrated in the following figure for $N = 2$

Step 2.4 : Determine the system function ($H_c(s)$) of Chebyshev-I CT filter (from the estimated poles)

From $N = 2$, the K can be mathematically expressed as following.

$$K = \begin{cases} 1 & , N \text{ odd} \\ \dfrac{1}{\sqrt{1+\varepsilon^2}} & , N \text{ even} \end{cases} \quad (15) \quad \rightarrow \quad K = \frac{1}{\sqrt{1+\varepsilon^2}}$$

$$\rightarrow \quad K = \frac{1}{\sqrt{1 + 2.0030^2}}$$

$$\rightarrow \quad K = 0.4467$$

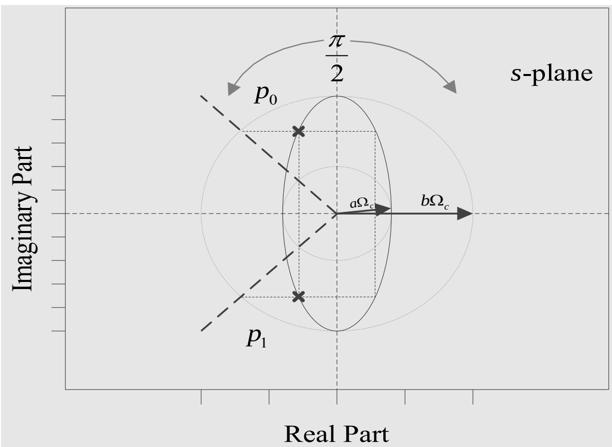

Figure 4. The 2 pole plot of the magnitude of squared function $H_c(s)H_c(-s)$ for 2th order Chebyshev-I filter

From $p_0 = (-0.1078 + j0.4572)$ and $p_1 = (-0.1078 - j0.4572)$, the system function ($H_c(s)$) of Chebyshev-I CT filter (from the estimated poles) can be mathematically expressed as:

$$H_c(s) = K \prod_{\text{LHP}} p_k \Big/ \prod_{\text{LHP}} (s - p_k) \quad (12)$$

$$\rightarrow \quad H_c(s) = \frac{(0.4467)\left[(-0.1078 + j0.4572)(-0.1078 - j0.4572)\right]}{(s - (-0.1078 + j0.4572))(s - (-0.1078 - j0.4572))}$$

$$\rightarrow \quad H_c(s) = \frac{(0.4467)[0.2207]}{(s^2 + 0.2156s + 0.2207)}$$

$$\rightarrow \quad H_c(s) = \frac{(0.4467)[0.2207]}{(s^2 + 0.2156s + 0.2207)}$$

$$\rightarrow \quad H_c(s) = \frac{0.0986}{(s^2 + 0.2156s + 0.2207)} \quad (16.6)$$

Step 3: Determine the system function $H_c(s)$ of continuous-time lowpass filter in the partial fraction expansion form, which can be mathematically expressed as following.

$$H_c(s) = \frac{0.0986}{(s^2 + 0.2156s + 0.2207)}$$

$$\rightarrow \quad H_c(s) = \frac{0.0986}{(s - (-0.1078 + j0.4572))(s - (-0.1078 - j0.4572))}$$

$$\rightarrow \quad H_c(s) = \frac{-j0.1078}{(s - (-0.1078 + j0.4572))} + \frac{j0.1078}{(s - (-0.1078 - j0.4572))} \quad (16.7)$$

Step 4: Determine the system function ($H(z)$) of the DT filter (from the system function ($H_c(s)$) of the Chebyshev-I CT filter) for $T_d = 1$

$$H_c(s) = \sum_{k=1}^{N} \frac{R_k}{(s - p_k)} \quad (12)$$

$$\rightarrow \quad H(z) = \left(\frac{-j0.1078}{\left(1 - e^{(-0.1078 + j0.4572)} z^{-1}\right)} + \frac{j0.1078}{\left(1 - e^{(-0.1078 - j0.4572)} z^{-1}\right)} \right)$$

$$\rightarrow \quad H(z) = \left(\frac{-j0.1078}{\left(1 - (0.8056 + j0.3964) z^{-1}\right)} + \frac{j0.1078}{\left(1 - (0.8056 - j0.3964) z^{-1}\right)} \right)$$

$$\rightarrow \quad H(z) = \frac{0.8056 z^{-1}}{\left(1 - 1.6111 z^{-1} + 0.8061 z^{-2}\right)} \quad (16.8)$$

The magnitude in dB, magnitude and phase of the frequency response of the analog filter $H_c(j\Omega)$ can be illustrated in Fig. 5(a), Fig. 5(b) and Fig. 5(c), respectively. Later, the magnitude in dB, magnitude and phase of the frequency response of the digital filter $H(e^{j\omega})$ can be illustrated in Fig. 6(a), Fig. 6(b) and Fig. 6(c), respectively. From these experimental simulation results, the impulse invariance concept can perfectly converse from the analog filter to the digital filter for magnitude perspective as shown in Fig. 6(a) and Fig. 7(a). Moreover, the phase of the frequency response of the digital filter, which is converted from analog filter, is severely distorted from original analog filter as shown in Fig. 6(c) and Fig. 7(c).

B. Experimental Simulation Results for Case 2

By using the impulse invariance techniques, design the Chebyshev-I IIR digital filter where the passband gain ($0 \le |\omega| \le 0.2\pi$) between 0 dB and -1 dB, and stopband ($0.3\pi \le |\omega| \le \pi$) has attenuation of -15 dB where $T_d = 1$. Sketch the magnitude in decibels (dB), the magnitude, the phase and the group delay of this frequency response of this Chebyshev-I IIR digital filter

The design of the DT IIR lowpass filter by using the impulse invariance can be expressed as following steps:

Step 1: Determine the continuous frequency of passband (Ω_p) and stopband (Ω_s) from the specification: ω_p, ω_s and T_d

$$\Omega_p = \frac{\omega_p}{T_d} \ (9) \quad \rightarrow \quad \Omega_p = \frac{0.2\pi}{1} = 0.2\pi \qquad (17.1)$$

and

$$\Omega_s = \frac{\omega_s}{T_d} \ (10) \quad \rightarrow \quad \Omega_s = \frac{0.3\pi}{1} = 0.3\pi \qquad (17.2)$$

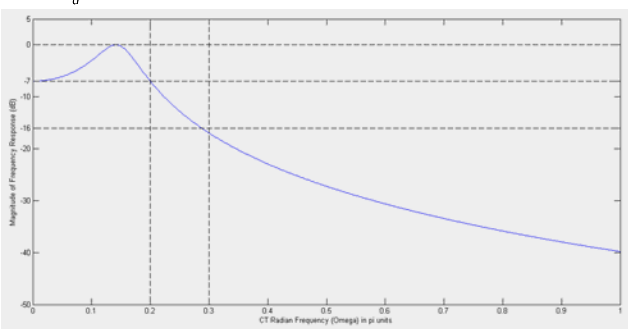

Figure 5 (a) The relationship between the magnitude in decibels (dB) of the frequency response of the analog filter, $|H_c(j\Omega)|$, and Ω.

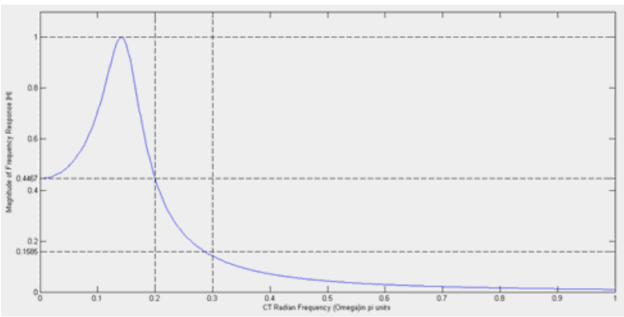

Figure 5 (b) The relationship between the magnitude of the frequency response of the analog filter, $|H_c(j\Omega)|$, and analog frequency Ω.

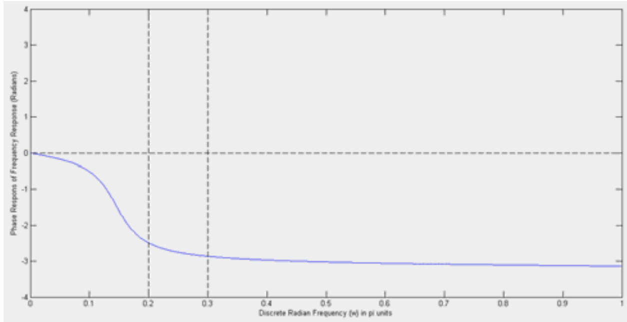

Figure 5 (c) The relationship between the phase of the frequency response of the analog filter, $\angle H_c(j\Omega)$, and analog frequency Ω.

Step 2: Determine the system function $H_c(s)$ of continuous-time low-pass filter (Chebyshev-I filter) from the specification: Ω_p, Ω_s, R_p and A_s (the detail of the continuous-time filter design is expressed in the preceding section)

Step 2.1 : Determine the order of Chebyshev-I low-pass filter, N, from the specification: R_p (passband ripple parameter) and A_s (stopband attenuation parameter)

$$\varepsilon = \sqrt{10^{\frac{-R_p}{10}} - 1} \quad (4) \quad \rightarrow \quad \varepsilon = \sqrt{10^{\frac{-(-1)}{10}} - 1}$$

$$\rightarrow \quad \varepsilon = 0.5088$$

$$A = 10^{\frac{-A_s}{20}} \quad (5) \quad \rightarrow \quad A = 10^{\frac{16}{20}}$$

$$\rightarrow \quad A = 6.3096$$

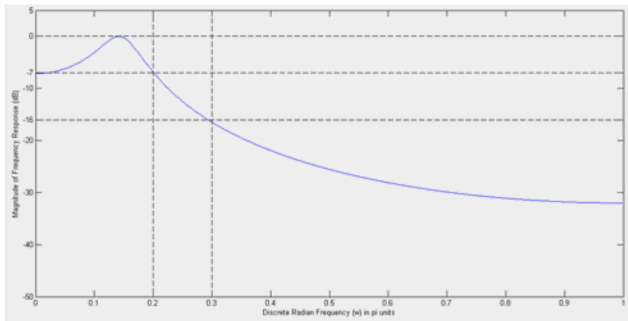

Figure 6 (a) The relationship between the magnitude in decibels (dB) of the frequency response of the digital filter, $20\log_{10}|H(e^{j\omega})|$, and ω.

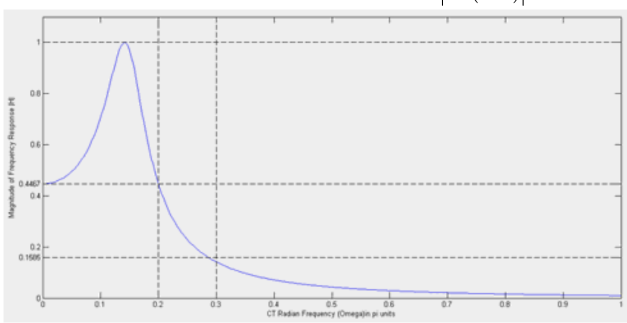

Figure 6 (b) The relationship between the magnitude of the frequency response of the digital filter, $20\log_{10}|H(e^{j\omega})|$, and digital frequency ω.

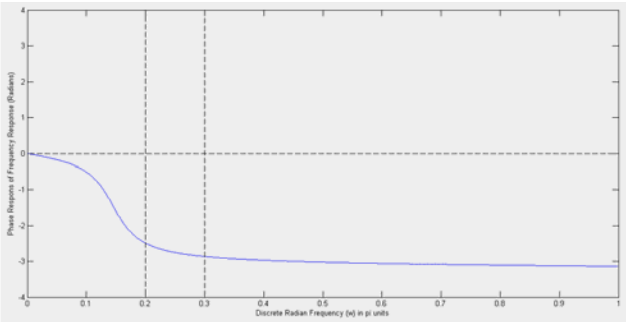

Figure 6 (c) The relationship between the phase of the frequency response of the digital filter, $\angle H\left(e^{j\omega}\right)$, and digital frequency ω.

$$g=\sqrt{\frac{\left(A^2-1\right)}{\varepsilon^2}} \quad (6) \quad \rightarrow \quad g=\sqrt{\frac{\left(6.3096^2-1\right)}{0.5088^2}}$$

$$\rightarrow \quad g=12.2442$$

$$\Omega_r=\frac{\Omega_s}{\Omega_p} \quad (7) \quad \rightarrow \quad \Omega_r=\frac{3\pi}{2\pi}$$

$$\rightarrow \quad \Omega_r=1.5$$

From g and Ω_r, the order of Chebyshev-I low-pass filter, N, can be mathematically expressed as following.

$$N=\left\lceil\frac{\log_{10}\left(g+\sqrt{g^2-1}\right)}{\log_{10}\left(\Omega_r+\sqrt{\Omega_r^2-1}\right)}\right\rceil \quad (8)$$

$$\rightarrow \quad N=\left\lceil\frac{\log_{10}\left(12.2442+\sqrt{12.2442^2-1}\right)}{\log_{10}\left(1.5+\sqrt{1.5^2-1}\right)}\right\rceil$$

$$\rightarrow \quad N=\lceil 3.3213\rceil$$

$$\rightarrow \quad N=4 \quad (17.3)$$

Step 2.2 : Determine the filter parameter Ω_c (or the cutoff frequency of the Chebyshev-I CT filter) from the specification: Ω_p.

$$\Omega_c=\Omega_p \quad (9) \quad \rightarrow \quad \Omega_c=2\pi=0.6283 \quad (17.4)$$

Step 2.3 : Determine the poles of the system function of the Chebyshev-I CT filter (from the filter parameter ε, N and Ω_c)

$$\alpha=\frac{1}{\varepsilon}+\sqrt{1+\frac{1}{\varepsilon^2}} \quad (10) \quad \rightarrow \alpha=\frac{1}{0.5088}+\sqrt{1+\frac{1}{0.5088^2}}$$

$$\rightarrow \quad \alpha=4.1706$$

$$a=0.5\left(\alpha^{1/N}-\alpha^{-1/N}\right) \quad (11)$$

$$\rightarrow \quad a=0.5\left(\left(4.1706\right)^{1/4}-\left(4.1706\right)^{-1/4}\right)$$

$$\rightarrow \quad a=0.3646$$

$$b=0.5\left(\alpha^{1/N}+\alpha^{-1/N}\right) \quad (12)$$

$$\rightarrow \quad b=0.5\left(\left(4.1706\right)^{1/4}+\left(4.1706\right)^{-1/4}\right)$$

$$\rightarrow \quad b=1.0644$$

Therefore, the poles (p_k) of this system function with stable property (The stable filter $H_c(s)$ can be defined by limiting poles in the left half-plain.) can be mathematically expressed as following.

$$p_k=\begin{pmatrix} \left(a\Omega_c\right)\cos\left(\frac{\pi}{2}+\frac{(2k+1)\pi}{2N}\right) \\ +j\left(b\Omega_c\right)\sin\left(\frac{\pi}{2}+\frac{(2k+1)\pi}{2N}\right)\end{pmatrix}, k=0,1,2,\ldots,(N-1) \quad (13)$$

$$\rightarrow \quad p_k=\begin{pmatrix} (0.3646\times 0.6283)\cos\left(\frac{\pi}{2}+\frac{(2k+1)\pi}{2(4)}\right) \\ +j(1.0644\times 0.6283)\sin\left(\frac{\pi}{2}+\frac{(2k+1)\pi}{2(4)}\right)\end{pmatrix}, k=0,1,2,3$$

$$\rightarrow \quad p_k=\begin{cases} p_0=\left((0.3646\times 0.6283)\cos\left(\frac{\pi}{2}+\frac{\pi}{8}\right)+j(1.0644\times 0.6283)\sin\left(\frac{\pi}{2}+\frac{\pi}{8}\right)\right) \\ ,p_1=\left((0.3646\times 0.6283)\cos\left(\frac{\pi}{2}+\frac{3\pi}{8}\right)+j(1.0644\times 0.6283)\sin\left(\frac{\pi}{2}+\frac{3\pi}{8}\right)\right) \\ ,p_2=\left((0.3646\times 0.6283)\cos\left(\frac{\pi}{2}+\frac{5\pi}{8}\right)+j(1.0644\times 0.6283)\sin\left(\frac{\pi}{2}+\frac{5\pi}{8}\right)\right) \\ ,p_3=\left((0.3646\times 0.6283)\cos\left(\frac{\pi}{2}+\frac{7\pi}{8}\right)+j(1.0644\times 0.6283)\sin\left(\frac{\pi}{2}+\frac{7\pi}{8}\right)\right) \end{cases}$$

$$\rightarrow \quad p_k=\begin{Bmatrix} p_0=(-0.0877+j0.6179), p_1=(-0.2116+j0.2559) \\ , p_2=(-0.2116-j0.2559), p_3=(-0.0877-j0.6179)\end{Bmatrix}$$

(17.5)

The pole plot of the system function of the Chebyshev-I CT filter is illustrated in the following figure for $N=4$

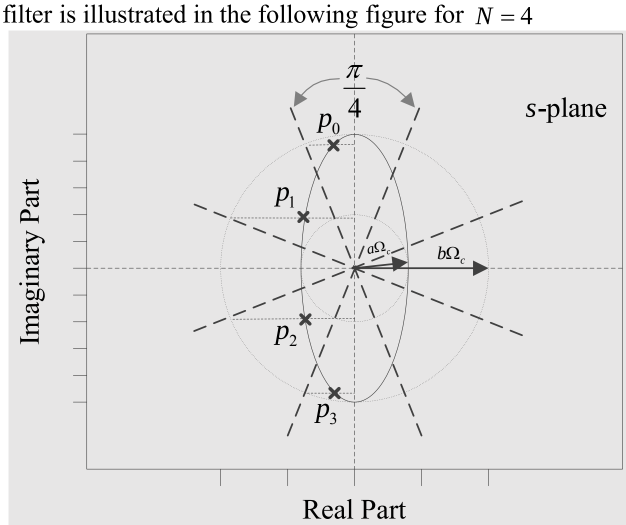

Figure 7. The 2 pole plot of the magnitude of squared function $H_c(s)H_c(-s)$ for 2[th] order Chebyshev-I filter.

Step 2.4 : Determine the system function ($H_c(s)$) of Chebyshev-I CT filter (from the estimated poles)

From $N=4$, the K can be mathematically expressed as following.

$$K=\begin{cases}1 & ,N \text{ odd}\\ \dfrac{1}{\sqrt{1+\varepsilon^2}} & ,N \text{ even}\end{cases} \quad (15) \quad \rightarrow \quad K=\frac{1}{\sqrt{1+\varepsilon^2}}$$

$$\rightarrow \quad K=\frac{1}{\sqrt{1+0.5088^2}}$$

$$\rightarrow \quad K=0.89125$$

From $p_0=(-0.0877+j0.6179)$, $p_1=(-0.2116+j0.2559)$, $p_2=(-0.2116-j0.2559)$ and $p_3=(-0.0877-j0.6179)$, the system function ($H_c(s)$) of Chebyshev-I CT filter (from the estimated poles) can be mathematically expressed as following.

$$H_c(s)=K\prod_{\text{LHP}} p_k \Big/ \prod_{\text{LHP}}(s-p_k) \quad (12)$$

$$\rightarrow \quad H_c(s)=\frac{(0.89125)\left[\begin{pmatrix}-0.0877\\+j0.6179\end{pmatrix}\begin{pmatrix}-0.2116\\+j0.2559\end{pmatrix}\begin{pmatrix}-0.2116\\-j0.2559\end{pmatrix}\begin{pmatrix}-0.0877\\-j0.6179\end{pmatrix}\right]}{\left(s-\begin{pmatrix}-0.0877\\+j0.6179\end{pmatrix}\right)\left(s-\begin{pmatrix}-0.2116\\+j0.2559\end{pmatrix}\right)\left(s-\begin{pmatrix}-0.2116\\-j0.2559\end{pmatrix}\right)\left(s-\begin{pmatrix}-0.0877\\-j0.6179\end{pmatrix}\right)}$$

$$\rightarrow \quad H_c(s)=\frac{(0.89125)\left[\begin{pmatrix}-0.0877\\+j0.6179\end{pmatrix}\begin{pmatrix}-0.2116\\+j0.2559\end{pmatrix}\begin{pmatrix}-0.2116\\-j0.2559\end{pmatrix}\begin{pmatrix}-0.0877\\-j0.6179\end{pmatrix}\right]}{(s^2+0.1754s+0.3895)(s^2+0.4234s+0.1103)}$$

$$\rightarrow \quad H_c(s)=\frac{(0.89125)\left[\begin{pmatrix}-0.2116\\+j0.2559\end{pmatrix}\begin{pmatrix}-0.2116\\-j0.2559\end{pmatrix}\right]\left[\begin{pmatrix}-0.0877\\+j0.6179\end{pmatrix}\begin{pmatrix}-0.0877\\-j0.6179\end{pmatrix}\right]}{(s^2+0.1754s+0.3895)(s^2+0.4234s+0.1103)}$$

$$\rightarrow \quad H_c(s)=\frac{0.89125\times0.1103\times0.3895}{\left[(s^2+0.1754s+0.3895)(s^2+0.4234s+0.1103)\right]}$$

$$\rightarrow \quad H_c(s)=\frac{0.03829}{\left[(s^2+0.1754s+0.3895)(s^2+0.4234s+0.1103)\right]}$$

$$\rightarrow \quad H_c(s)=\frac{0.03829}{(s^4+0.5987s^3+0.5740s^2+0.1842s+0.043)} \quad (17.6)$$

Step 3: Determine the system function $H_c(s)$ of continuous-time lowpass filter in the partial fraction expansion form, which can be mathematically expressed as following.

$$H_c(s)=\frac{0.03829}{(s^4+0.5987s^3+0.5740s^2+0.1842s+0.043)}$$

$$\rightarrow \quad H_c(s)=\frac{0.03829}{\left(s-\begin{pmatrix}-0.0877\\+j0.6179\end{pmatrix}\right)\left(s-\begin{pmatrix}-0.2116\\+j0.2559\end{pmatrix}\right)\left(s-\begin{pmatrix}-0.2116\\-j0.2559\end{pmatrix}\right)\left(s-\begin{pmatrix}-0.0877\\-j0.6179\end{pmatrix}\right)}$$

$$\rightarrow \quad H_c(s)=\begin{pmatrix}\dfrac{-0.0415+j0.0817}{(s-(-0.0877+j0.6179))}+\dfrac{0.0415-j0.2177}{(s-(-0.2116+j0.2559))}\\+\dfrac{0.0415+j0.2177}{(s-(-0.2116-j0.2559))}+\dfrac{-0.0415-j0.0817}{(s-(-0.0877-j0.6179))}\end{pmatrix} \quad (17.7)$$

Step 4: Determine the system function ($H(z)$) of the DT filter (from the system function ($H_c(s)$) of the Chebyshev-I CT filter) for $T_d=1$:

$$H_c(s)=\sum_{k=1}^{N}\frac{R_k}{(s-p_k)} \quad (12)$$

$$\rightarrow \quad H(z)=\begin{pmatrix}\dfrac{-0.0415+j0.0817}{\left(1-e^{(-0.0877+j0.6179)}z^{-1}\right)}+\dfrac{0.0415-j0.2177}{\left(1-e^{(-0.2116+j0.2559)}z^{-1}\right)}\\+\dfrac{0.0415+j0.2177}{\left(1-e^{(-0.2116-j0.2559)}z^{-1}\right)}+\dfrac{-0.0415-j0.0817}{\left(1-e^{(-0.0877-j0.6179)}z^{-1}\right)}\end{pmatrix}$$

$$\rightarrow \quad H(z)=\left(\frac{-0.0830+0.0247z^{-1}}{(1-1.4933z^{-1}-0.8391z^{-2})}+\frac{-0.0830+0.0242z^{-1}}{(1-1.5659z^{-1}-0.6549z^{-2})}\right)$$

$$\rightarrow \quad H(z)=\frac{0.0054+0.0181z^{-1}-0.0040z^{-3}}{(1-3.0591z^{-1}+3.8323z^{-2}-0.2918z^{-3}+0.5495z^{-4})} \quad (17.8)$$

The magnitude in dB, magnitude and phase of the frequency response of the analog filter $H_c(j\Omega)$ can be illustrated in Fig. 8(a), Fig. 8(b) and Fig. 8(c), respectively (after References). Later, the magnitude in dB, magnitude and phase of the frequency response of the digital filter $H(e^{j\omega})$ can be illustrated in Fig. 9(a), Fig. 9(b) and Fig. 9(c), respectively. From these experimental simulation results, the impulse invariance concept can perfectly converse from the analog filter to the digital filter for magnitude perspective as shown in Fig. 8(a) and Fig. 9(a). Moreover, the phase of the frequency response of the digital filter, which is converted from analog filter, is slightly distorted from original analog filter as shown in Fig. 8(c) and Fig. 9(c).

V. CONCLUSION OF EXPERIMENTAL SIMULATION

This scientific paper wholly investigates the capacity of the digital IIR filter design technique using Butterworth concept and impulse invariance concept. In this simulated experiment, we demonstrates the filter desiring procedure in both mathematical and computer simulation perspective. From these result, this design technique has good performance for magnitude response requirements nevertheless this design technique has poor performance for phase response requirements, especially at high frequency.

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MORE RESULTS

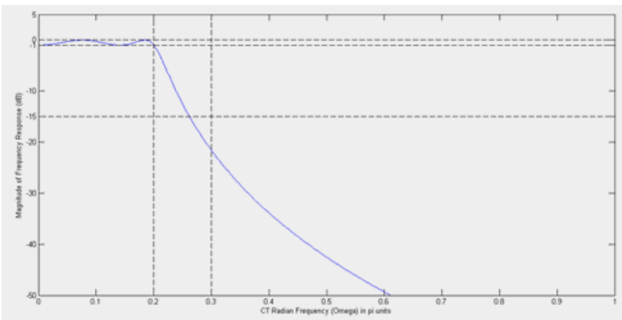

Figure 8 (a) The relationship between the magnitude in decibels (dB) of the frequency response of the analog filter, $20\log_{10}\left|H_c\left(j\Omega\right)\right|$, and Ω.

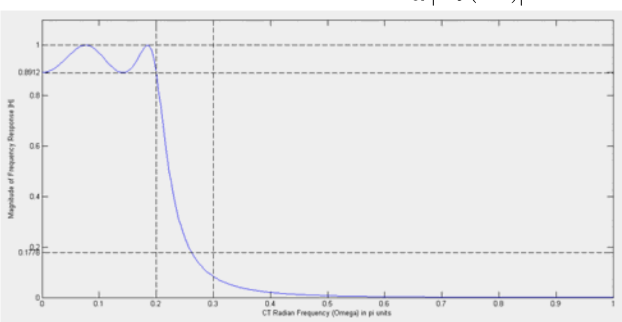

Figure 8 (b) The relationship between the magnitude of the frequency response of the analog filter, $\left|H_c\left(j\Omega\right)\right|$, and analog frequency Ω.

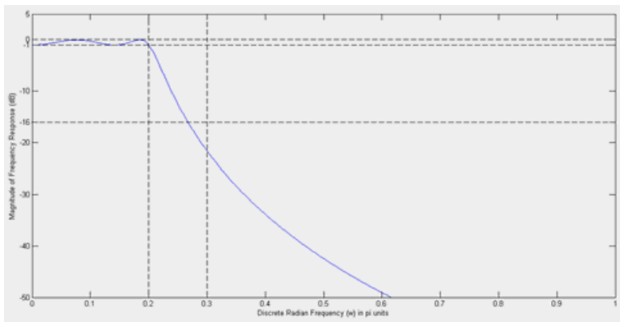

Figure 8 (c) The relationship between the phase of the frequency response of the analog filter, $\angle H_c\left(j\Omega\right)$, and analog frequency Ω.

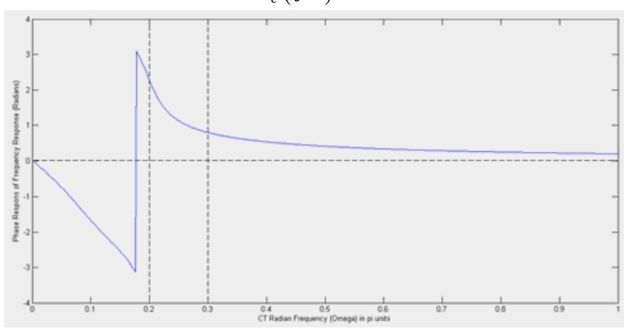

Figure 9 (a) The relationship between the magnitude in decibels (dB) of the frequency response of the digital filter, $20\log_{10}\left|H\left(e^{j\omega}\right)\right|$, and ω.

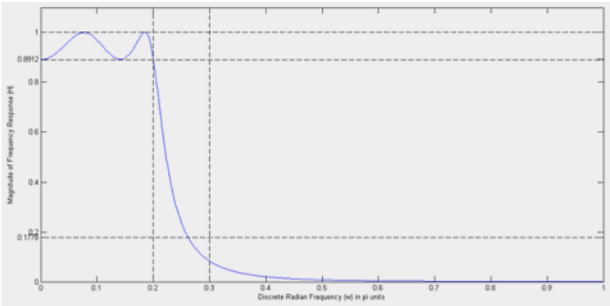

Figure 9 (b) The relationship between the magnitude of the frequency response of the digital filter, $\left|H\left(e^{j\omega}\right)\right|$, and digital frequency ω.

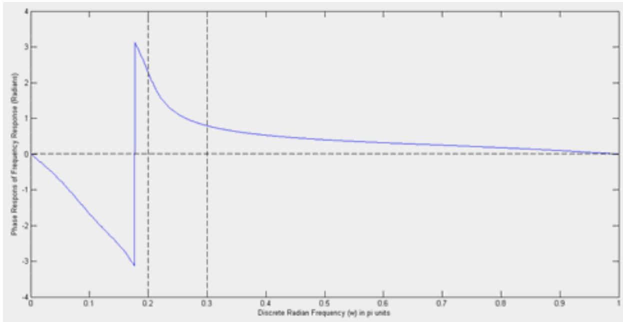

Figure 9 (c) The relationship between the phase of the frequency response of the digital filter, $\angle H\left(e^{j\omega}\right)$, and digital frequency ω.