Optimization Heuristics for Dynamic Vehicle Routing Problems

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***Abstract* - The management of goods delivery is becoming very important. The on time delivery is a critical criterion taking into account customers point of view. But the delivery company must also pay attention to the economic considerations. There are many variations on this issue, but all of them are of great computational complexity. It means that the exact solutions are unavailable for large size problem. The paper proposes the Surrogate Method for the Dynamic Vehicle Routing Problem (DVRP). The aim of DVRP is to find a set of routes to serve multiple customers while the travelling time between point to point may vary during the process. The aim is to schedule the vehicle routes minimizing the number of the required vehicles and the completion time. The presented approach uses some common assumptions but different optimization method. Finally, the proposed heuristic is compared with the genetic algorithm.**

Keywords - Delivery problem, vehicle routing problem, heuristics.

I. INTRODUCTION

Vehicle routing is obviously significant in the areas of logistics, transportation and related industries. Thus it has attracted a large number of researchers and practitioners from various sectors, which is often referred as VRP (Vehicle Routing Problem). Classical VRP is a well-known combinatorial optimization problem and has been intensively investigated. VRP involves a set of customers geographically distributed at different locations and a fleet of vehicles. The goal is serving all customers at minimal cost (e.g. travelling distance, time, fuel etc.) while respecting all constraints. Due to the nature of real-world vehicle routing problems, several variants of VRP have been formulated. Each of these VRP variant accommodates certain constraints and factors to reflect one type of real-life scenarios.

Well-known VRP variants include: i) CVRP, the Capacitated Vehicle Routing Problem which has vehicle capacity as a hard constraint; ii) VRPTW, the Vehicle Routing Problem with Time Windows where each customer can only accept services/deliveries during a predefined time window; and iii) VRPMT, the Vehicle Routing Problem with Multiple Trips, where a vehicle can take more than one trip/task.

In these vehicle routing scenarios, the travel time from one point to another is a constant. In VRP, a fleet of vehicles must service the demands of customers. A vehicle begins and ends its route at the same depot and the sum of the demands of the customers on a route cannot exceed a vehicles capacity. A customer must have all of its demand delivered at one time by a single vehicle. The objective is to minimize the total distance traveled by the fleet. It is still encountered in our days, mainly in the domain of logistics and transport. In the VRP, m vehicles (vi), with identical capacities (Q), initially located at a central depot (v0), are to deliver discrete quantities of goods (qi) to n customers, which are geographically diffused around the central depot. Concurrently, the aim of the VRP, beyond serving customers, is to minimize the travelled distance.

Due to the difficulty the VRP presents and because of its practical applications, many models have been created for solving the problem and many variants of the basic VRP have been compiled, with different parameters, leading to a different structure of the basic VRP. Firstly, the classical VRP is equivalent to the Capacitated VRP (CVRP) in which, the capacity of the vehicle must not be exceeded (see [3], [6], [9], [14]). However, there is another possibility that the vehicles do not have the same capacities which leads to the Heterogeneous Fleet VRP (HVRP).

Another important variant which was created a decade after the classical VRP was the Multi-Depot VRP (MDVRP) in which, the company has several depots from which it can serve its customers, while the objective is still to service all customers and minimize the number of vehicles and distance travelled (see [23]). At the same period, the Stochastic VRP (SVRP) was created, where, the customers, the demand into account time is the Time Dependent VRP (TDVRP) in which travel times change as time passes. The reason why this happens is due to traffic congestion. The factors which affect travel times are: (1) the location, and (2) the time of the day.

During the last decade, many researchers have tried, while solving the VRP, to minimize carbon (or fuel emissions) as carbon dioxide (CO_2) emitted by trucks is the main greenhouse gas. In addition, the Green VRP has been strengthened due to the technical developments and the road traffic information which allows planning vehicle routes and schedules and taking time-varying speeds into account (5, 7, 8). In the same field belongs the Hybrid VRP where vehicles can work both electrically and with petroleum-based fuel.

The Dynamic Vehicle Routing Problem (DVRP) is a complex variation of classical Vehicle Routing Problem (VRP). The aim of DVRP is to find a set of routes to serve multiple customers at minimal total travelling cost while the travelling time between point to point may vary during the process because of factors like traffic congestion. To effectively handle DVRP, a good algorithm should be able to adjust itself to the changes and continuously search for the best solution under dynamic environments. Because of this dynamic nature of DVRP, evolutionary algorithms (EAs) appear highly appropriate for DVRP as they search in a parallel manner with a population of solutions. Solutions scattered over the search space can better capture the dynamic changes. Solutions for new changes are not built from scratch as they can inherit problem-specific knowledge from parent solutions. However, the performance of EA is highly dependent on the utilized configuration

Travel-time plays an important role in the distribution of the perishable goods, since its fluctuations may extend the time that the goods spend on the vehicles. Different representations of the fluctuations of the travel-times between the customers have been reported and different extensions of VRP have been proposed to address the fluctuations in travel-times. Routing problems with stochastic travel-times are presented by ([15], [16], [17]). In [2], they presented a TSP in which they have considered a zone in the city center with traffic jams in the afternoon and show how simulated annealing and threshold accepting algorithms are able to handle such time-dependent problems. In [24] Park presented the time-dependent VRP in which the travel speed between two locations depends on the route and the time of the day. They proposed a model for estimating the time varying travel speed. In [18], they proposed a time-dependent model for the VRPTW. The model that they developed is based on time-dependent travel speeds and satisfies the first-in-first-out (FIFO) property. They extended the tabu search heuristic to solve the problem and showed that the time-dependent model provides substantial improvements over a model based on fixed travel-times.

Classical VRP and its variants are known as -hard in terms of the problem complexity. That means finding the optimal solution could be impractical for VRP instance of reasonable size due to the prohibitive computational resource required. Exact methods, which guarantee optimal solutions, are only advisable to be used on small instances. In reality small instances have little practical values as real world problems often are large in size. Thus, meta-heuristic algorithms are better alternatives in these scenarios, as they can often generate solutions of good quality within an acceptable amount of time. This kind of method offers no guarantee of optimality but high application value as the good solutions generated by them are often not far from the optimal solutions. Typical metaheuristic algorithms include Tabu search, simulated annealing, evolutionary algorithms, ant colony and variable neighborhood algorithms (see [13], [19], [20], [27], [28]. In reality most of logistics and transportation problems are dynamic by nature. Only limited information is available at the beginning of a trip. New information arrives over time. For example, a new order from customer may appear whilst vehicles are already on road serving customers. Another important factor is traffic condition, which may vary dramatically in different time of a same day, or a same time in different days. It is difficult to foretell the level of traffic congestion (see [1]). This leads to a challenging and realistic dynamic variant of the VRP, which is denoted as DVRP, referring the vehicle routing problem with traffic congestion. Dynamic order occurs much less often in comparison. In DVRP the exact travel time between customers is not known in advance and subject to the level of the traffic congestion on the path, meaning that after generating a set routing plans and after the vehicles have left the depot to serve customers, the travel time between customers may change. When a change occurs, the optimization algorithm should adapt to it and find new solutions at minimal cost. However, DVRP is relatively unexplored despite its theoretical importance and practical values.

The following section of the paper contain short problem description and its mathematical formulation, then the solution method is described, results for a case study are presented and finally some conclusions are discussed.

II. PROBLEM DESCRIPTION

In this section, we first formally describe the classical VRP, then the dynamic VRP variant. In classical VRP, there is a set of geographically spread customers with known demands and a fleet of vehicles of fixed capacity. VRP can be formulated as a mathematical model as follows. Let G(V,E) be a complete directed graph where $V = \{v_0, v_1, ..., v_n\}$ is a set of nodes. Node v_0 is the depot which has m vehicles and nodes $v_1; ...; v_n$ represent a set of customers. Vehicles have identical capacity, $Q = Q_1, ..., Q_m$. E represents a set of edges connecting customers v_i and v_j , $E_{i,j} = \{(v_i , v_j) \; v_i , v_j \in V, \; i \leq j\}$. Each edge Ei,j has a non-negative value which is the cost e.g. the travel time between v_i and v_j. The cost is defined by a matrix $C = (c_{i,j})$. An entry $c_{i,j}$ of the matrix C represents the shortest path between customers v_i and v_j . Each customer v_i is associated with a value representing q_i goods to be delivered or picked at this customer. Each delivery has a service time t_i . The goal of DVRP optimization is to find a set of vehicle routes to serve all customers at minimal cost while satisfying the following constraints:

- All vehicles must start and end their routes, R_1, ..., R_m, at the depot v_0;
- The total demand assigned for each vehicle should not exceed the vehicle capacity;
- Each customer is visited only once in the delivery plan;
- The total duration of each route should not exceed the given global upper bound;

In real-life situations, the travel time between nodes depends on traffic condition of the current road network. Traffic could vary significantly depending on the time of the day. For example, the travel time during rush hour would be multiple times higher than the time travelling at midnight, see Fig.1. For this reason, we consider time dependent travel times that vary respect to the time slice of the day considering the traffic condition of the network and possible evolution on the basis of historical data.

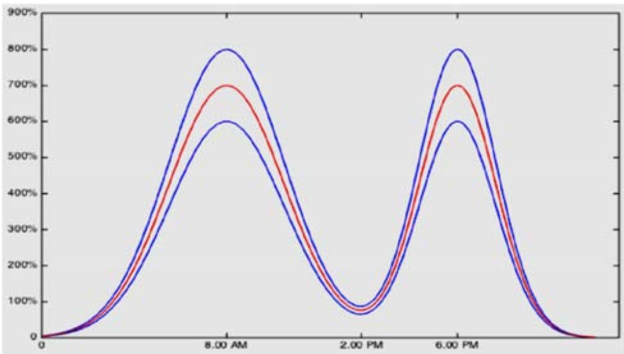

Fig. 1. Trend of traffic during the day

For each pair (v_i, v_j) a set $S_{i,j}$ of shortest paths from v_i position to v_j is calculated, one for each considered time slice. Note that, when constructing the shortest path, the release date of v_i and the traffic condition in a specific time slice effect its shortest path to reach v_j.

The objective function takes into account both the duration of delivery process and the number of vehicles. Duration time of the delivery for a vehicle is defined as the time required for visiting all the planned nodes on its route, it is measured from the departure from the source point to the end of the service of the last node on the route. This time includes also the nodes service time for all planned nodes for each vehicle route. Only the nodes service time is time independent. This completion time for delivery (C_D) is compared with the established time limit (T_{MAX}), and the following rules are assumed:

- if C_D is less than the limit T_{MAX}, then the objective function is equal to 1 (satisfying time);
- if C_D is longer by more than half of the established limit, the function is equal to 0 (unsatisfactory time);
- in the other cases the objective function is linearly dependent on C_D.

For this purpose these rules are summarized as:

$$OF_T(C_D) = \begin{cases} 1, C_D \leq T_{MAX} \\ \frac{1.5T_{MAX}-T_{MAX}}{0.5T_{MAX}}, T_{MAX} \leq C_D \leq 1.5T_{MAX} \\ 0, C_D \leq 1.5T_{MAX} \end{cases} \quad (1)$$

Considering the number of vehicles involved N for delivery and the maximum number of vehicles that are potentially available NMAX the objective function becomes:

$$OF_N = \frac{N_{MAX} - N}{N_{MAX}} \quad (2)$$

The total objective function is calculated by multiplying these two elements, and the problems becomes:

$$MAX\{OF_T(C_D) * \frac{N_{MAX} - N}{N_{MAX}}\} \quad (3)$$

The scheduling of deliveries and the routing of vehicles in urban areas is affected by traffic conditions which have a significant impact on travel time and consequently on delivery efficiency and customer service. A solution is a set of routes one for each vehicle and all costumers are satisfied. Each solution variant is evaluated, where the assessment takes into account both the duration of the delivery process and the number of vehicles involved. Duration time of the delivery for a vehicle is defined as the time required for visiting all the planned nodes on its route, it is measured from the departure from the source point to the end of the service of the last node on the route.

The presented problem is similar to the travelling salesman problem: the shortest path which visits the given set of the graph nodes should be found. If all the nodes to be visited are numbered in a range from 1 to N (where N denotes the total number of customers) and the starting point is marked as 0, each series of such numbers is a valid schedule of visiting the nodes:

$$(0, K_1, K_2; \ldots, K_N) \quad (4)$$

where ki number of a node, which will be serviced as i-th.

III. OPTIMIZATION HEURISTIC: THE SURROGATE METHOD

The Surrogate Method presents an iterative structure that, for each cycle, transforms the discrete problem with decision variable vector Z, in an optimization problem with continuous decision variable vector ρ, by relaxing the integer constraints. The latter problem is denoted as Surrogate. Subsequently, the gradient estimate of the objective function (∇OF), which is used to update the solution, is computed in the discrete field. The transition from the discrete problem to the continuous problem occurs at each cycle of the algorithm and the update of the

DOI 10.5013/IJSSST.a.20.S1.16

ISSN: 1473-804x online, 1473-8031 print

surrogate solution is obtained by considering the variation of the objective function computed in the discrete state.

Vector Z is an integer n-dimensional decision vector where each component denotes a costumer, the order of the vector component represents the route of vehicles. A route is composed by the component of the vector that satisfies all the constrains. *OF(Z)* is the cost of the solution when the state is *Z*. The integer capacity constraint is relaxed and a resulting surrogate problem is obtained. When and if a solution of the surrogate problem ρ^* is obtained, it is possible to map it through the transformation function *f* into a discrete point $Z = f(\rho^*)$, which is the solution of the discrete problem. Function f selects the integer z_k that minimizes the difference $|z - \rho_k|$.

Note however that the sequence $\{\rho_k\}$, $k = 1, 2, \ldots$, generated by an iterative scheme to solve the relaxed problem, consists of real-valued solutions which are unfeasible for the discrete problem. Thus, a key feature of the Surrogate algorithm is that at each iteration *k* of the scheme, a discrete state z_k is updated through the function *f*: $z_k = f(\rho_k)$ as ρ_k.

This has two advantages:

- the cost of the discrete system is cyclically adjusted (in contrast to an adjustment that would only be possible at the end of the Surrogate optimization process);
- the scheme combines the advantages of stochastic approximation type of the algorithm with the ability to obtain sensitivity estimates with respect to discrete decision variables.

For this kind of problem the calculation for the transformation function *f* is not a simple task. It is possible that the integer vector z_k obtained from the current continuous vector ρ_k may have some repeated nodes, but a solution is considered admissible if it contains all the nodes of the graph only once. For this reason all the repeated elements are substituting with the missing one, this procedure is not unique. For example if one element is repeated two times, the set of admissible solution As is composed by two vectors (i.e. for six nodes (n=6), if z_k = (1,6,1,4,5,2) the set of admissible solution is A_s = {(1, 6, 3, 4, 5, 2),(3, 6, 1, 4, 5, 2)}). Instead if one element is repeated three times, the set of admissible solution is composed by six vectors (i.e. if z_k = (2,1,3,4,4,4) then A_s = {(2, 1, 3, 4, 5, 6), (2, 1, 3, 4, 6, 5), (2, 1, 3, 5, 4, 6), (2, 1, 3, 5, 6, 4), (2, 1, 3, 6, 4, 5),(2, 1, 3, 6, 5, 4)}), and so on.

This set of admissible solution A_s is used both to calculate the discrete vector z_k and to calculate the selection set $S(\rho_k)$. To reduce the calculation time a sub set (*SA*) of fixed dimension (*D*) for the admissible solutions is considered ($S_A \subset A_s$). Obviously, this sub set is not unique and a methodology for selecting the *D* vectors is under study. In this study (*SA*) is obtained by the first (*D*) changes of z_k. The transformation function *f* gives the vector in *SA* that optimizes the objective function. S_A also contributes for calculating the selection set $S(\rho_k)$. This selection set is formed using the steps reported in Fig.2 plus the vectors in sub set of A_s.

IV. RESULTS ANALYSIS

At this point, a serious problem arises for the service provider, how to deliver the desired goods to a large number of customers waiting for the delivery at various points of the city, at a specified time, with at low cost as possible. The presented case study makes use of the data published by [22].

Steps of the Surrogate Method SM

- Initialize $\rho_0 = z_0$ satisfying constraints
(ρ_0 is a continuous vector, z_0 is a discrete vector, both of dimension $M+1$)
- Initialize $\rho^* = \rho_0$, $k^* = k$
(ρ^* is the optimal solution of the continuous problem)
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(ρ^* is the optimal solution of the continuous problem)
- Initialize $h = 0$
(number of consecutive iterations in which the solution does not improve)
while $((k \le K) \vee (h \le H))$ **do**
Form the selection set $S(\rho_k)$
- Initialize I= $\{1, ..., M\}$ and $v = \rho - \lfloor \rho \rfloor$
while $(I \neq \emptyset)$ **do**
$i = \arg\min_{j \in I}(v[j])$
(the component $v[i]$ is the decimal part of the $\rho[i]$ component)
$W_i = \sum_{j \in I} e_j$
$v = v - y[i]W_i$
$I = I \setminus \{i\}$
end
end
- $S(\rho_k) = \{W_i - \lfloor \rho \rfloor, i = 0, .., M\}$
Transform the continuous problem to the discrete problem
- $z_k = f(\rho_k) = \arg\min_{z \in Z} \|z - \rho_k\|$
Transformation function f
Gradient estimate
- $\nabla OF(\rho_k) = [\nabla_1 OF, ..., \nabla_M OF]^T$
(OF is the Objective function,
where $\nabla_j OF(\rho_k) = OF(p) - OF(q)$
where k satisfies $p - q = e_j$ and $p, q \in S(\rho_k)$,
being e_j the versor with j-th component equal to 1)
Update state
- $\rho_{k+1} = f[\rho_k - \eta_k \nabla OF(\rho_k)]$.
Optimal solution update
if $OF(\rho_k) \le OF(\rho^*)$ **then**
$\rho^* = \rho_k$
$h = 0$
end
else
$h = h + 1$
end
Return the optimal solution z^*
- **Return** $z^* = f(\rho^*)$.
- **Return** $z^* = f(\rho^*)$.

Fig. 2. Steps of SM

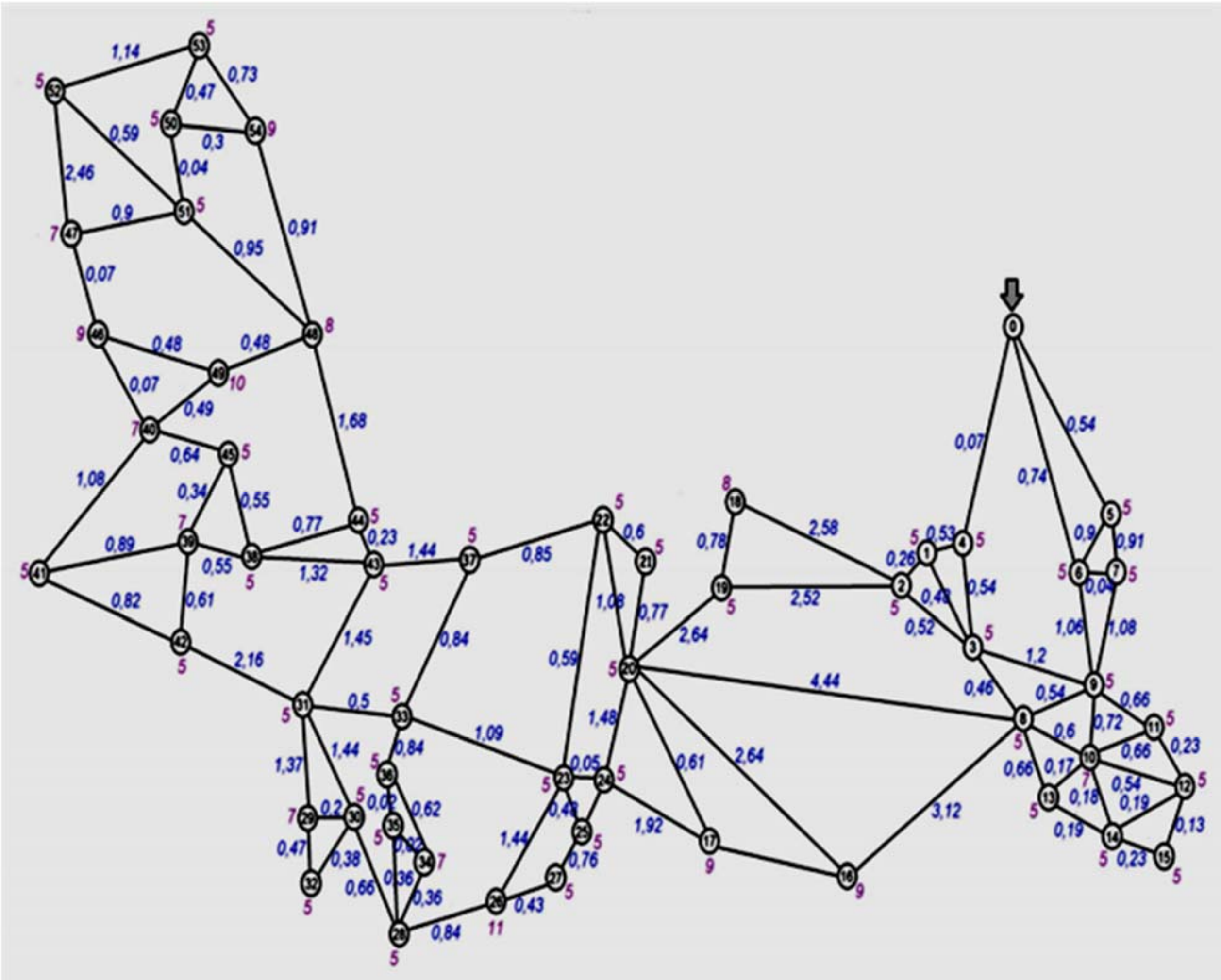

Fig. 3. Test Network for a time slice

In Fig.3, the graph presents the simplified structure of the transportation network and the layout of customers and the location of the supplier company. There is a driving time assigned to each edge and a service time assigned to each node. It is represents the network and the time for a specific time slice. Given the stochastic nature of the problem, it was assumed that a solution is the average on 20 runs of optimization procedure. T_{MAX} is equal to 40 minutes and N_{MAX} is equal to 30.

As was mentioned in the introduction section the considered problem is computationally very complex, therefore the genetic algorithm (GA) is considered and compared with the surrogate method (SM).

A. Genetic Heuristic

This heuristic is a standard genetic algorithm. The initial solution is generated considering first the number of vehicles in service is randomly drawn, then the nodes to be serviced are sequentially assigned to vehicles. After generating the initial population by a random assignment, a cycle, composed by the following steps, runs till the stop criteria are met.

1. For each solution evaluate the objective function.
2. Select a set of best solutions on the basis of step 2 and save them.
3. Combine them to create a new population.

When the cycle stops the best solution is given in output.

The GA and the SM try to minimize number of required vehicles and the time needed for service. Some preliminary results are reported The trends of the two algorithms for the different runs is reported in Fig. 4. The variation respected to the run is very limited for both algorithms.

In table I, a comparison between heuristics is reported, considering also a variation for the SM. In SM$*$ the constrain on the completion time for delivery is relaxed. The table I shows the values of the objective function, the number of utilized vehicles and the duration of the completion time. The surrogate method finds a solution with minor competition time.

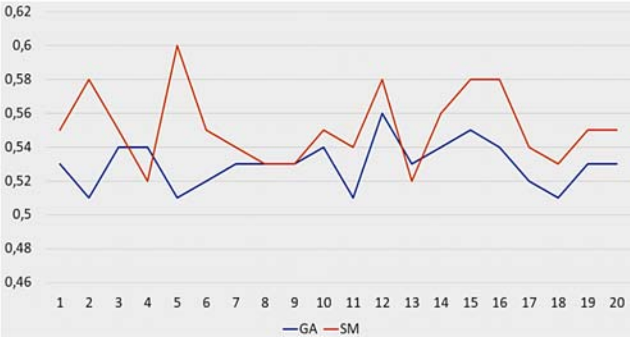

Fig. 4. Trend of the objective function

TABLE I. COMPARISON BETWEEN SM AND GA

HEURISTIC	OF	N	C_D
SM	0.55	14	38
SM*	0.53	12	42
GA	0.53	14	39,43

When the constrain on completion time is relaxed the surrogate method finds a solution with minor number of utilized vehicles but with bigger competition time. This preliminary results seams to demonstrate also for this type of problem the ability of surrogate method to find optimal solution for complex problem.

V. CONCLUSION

Scheduling deliveries in a large catering company may require the use of quick methods for determining the routes of vehicles. This study is at begging phase and a more complete analysis is under study. The building of the selection set S(ρk) is in under study. For this kind of problem the methodology illustrated by the classical surrogate method is unmissable, but it is fundamental for the fast convergence of the algorithm to an optimal solution.

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