

## A Numerical Model for Salinity Intrusion Measurement and Control Based on MacCormack Finite Difference Method and Cubic Spline Interpolation

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**Abstract** - The purpose of this research was to develop a numerical model of one-dimensional advection-diffusion equation for estimating salinity level in Lower Chao Phraya River, Thailand. The main numerical technique was MacCormack finite-difference scheme with initial and boundary conditions approximated by cubic spline interpolation. The estimates matched satisfactorily with actual measurement data from several salinity monitoring stations along the river. The final model was first used by Thailand's Metropolitan Waterworks Authority for planning salinity control in 2018.

**Keywords** - salinity intrusion, MacCormack scheme, cubic spline interpolation, Chao Phraya River

### I. INTRODUCTION

Freshwater is one of the most important natural resources for the livelihood of humans, animals, and plants. In Thailand, tap water produced from freshwater along Chao Phraya River is consumed by millions of people in Bangkok metropolitan. Good quality freshwater is also depended on by agricultural and manufacturing industries situated near the river. Recently, seawater intrusion from the estuary into lower Chao Phraya River has become more aggressive, creating problems with tap water production and low water quality for said industries. To counter the intrusion, freshwater needed to be released from an upriver dam. However, during dry season, the amount of freshwater held by the dam is low, and there is a need to strictly control the amount released for countering the intrusion. In planning out a good control strategy, fast and accurate salinity level readings are necessary, which may not be readily and timely supplied by various salinity monitoring

stations along the river. Therefore, a numerical model that can accurately and immediately provide good salinity level estimates at key points along the river is in dire need. Our work is an attempt to solve this issue.

Chao Phraya River is the major river in the central region of Thailand starting from a confluence of four rivers at Pak Nam Pho ( $14.23848^{\circ}N, 100.575364^{\circ}E$ ), flowing pass several provinces including Bangkok and down to the estuary ( $13.58^{\circ} - 15.67^{\circ}N, 100.10^{\circ} - 101.00^{\circ}E$ ) to the Gulf of Thailand for the total distance of 300 kilometers.

Several salinity monitoring stations have been erected along the river. Samlae pumping station that acts as a salinity monitoring station and a pumping station for tap water production for Bangkok Metropolitan is 96 km upriver from the estuary (see Figure 1). The actual measurement data for our reference was taken from this station [1].

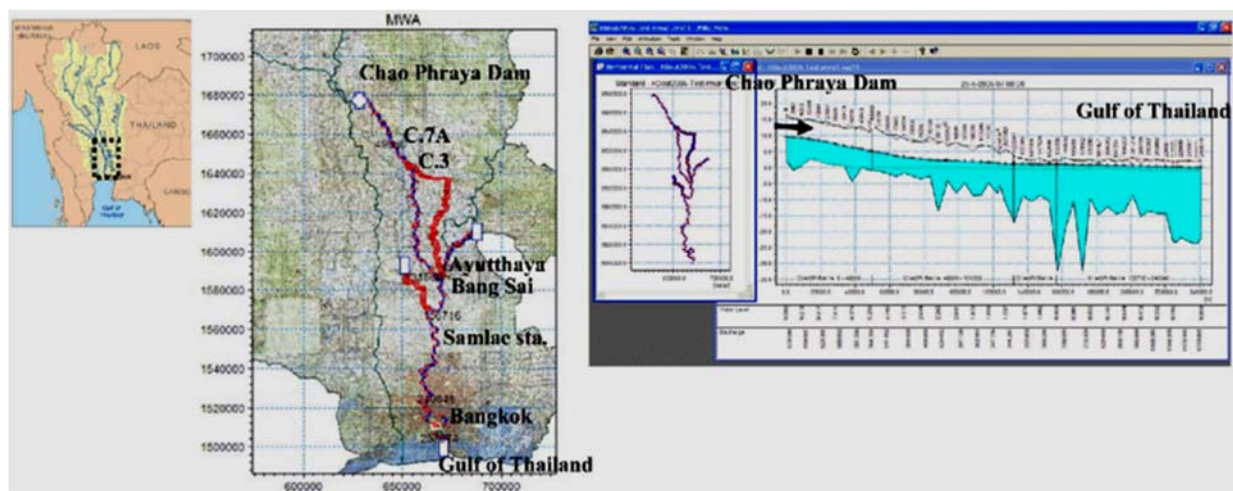


Figure 1. Lower Chao Phraya River and levels of seawater intrusion along the river [2]

Recently, the use of mathematical models to calculate natural phenomena has become more popular due to in-depth findings of natural phenomena processes and advancement in computer hardware and software. Solutions of complex system of equations representing natural processes can be approximated quickly with new numerical techniques. Many researchers have used water-quality models to approximate salinity level in rivers. Intaboot and Taesombat [3] investigated salinity intrusion in the area of the Tha Chin estuarine. They used a mathematical model called “MIKE-HD/AD” to predict that the maximum distance of salinity intrusion would reach 55 kilometers further up the river than it was in 2015 and provided a suitable freshwater flow rate to counter the intrusion of about 20-40 cubic meters per second. A higher flow rate would not help pushing salinity out any better. As another example, A. Owen [4] has reported a development of numerical modeling of salinity’s advective transport in coastal areas, associated with artificial diffusion using upstream differences. This particular model can be used small grids of two to four kilometers for tidal problems, while larger grids are used for non-tidal problems. Ralston et al. [5] applied Hansen and Rattray equations to simulate the distribution of salinity and vertical exchange flow along the estuary of the Hudson River. N. Pochai [6] used an improved finite difference scheme to solve an advection-dispersion-reaction equation (ADRE) that takes into account the effect of stream’s nonuniform water flow. Two mathematical models, a hydrodynamic model and an advection-dispersion-reaction model were used to simulate pollution from sewage effluent. Li and Jackson [7] reported another case of stream water quality modeling using simple modifications of MacCormack and Saul’yev schemes to solve dynamic one-dimensional advection–dispersion–reaction equations (ADRE). The prediction accuracy was better than the original schemes without major loss of computational efficiency.

Several researchers have used Cubic Spline interpolation technique to approximate data related to salinity intrusion and level in rivers. R.M. Corless [8] detailed a connection between cubic splines and a popular compact finite difference formula, claiming that a compact approach with nonuniform meshes provides some advantages while the approach with uniform meshes helps treat edge effects. Ariffin and Karim [9] used two types of cubic spline functions—cubic spline interpolation with  $C^2$  continuity and Piecewise Cubic Hermite Spline (PCHIP) with  $C^1$  continuity for interpolating data. Both cubic splines were numerically compared with each other as well as with linear spline and achieved good results. In a paper by Ahmad et al. [10], a mathematical model of teeth of a child was compared with cubic spline method to see which one produced a minimum error in showing the general form of normal teeth and created a better view of the nature of orthodontic wires. The cubic spline method showed a more

symmetrical teeth’s shape; the curves of a tooth were more symmetrical than the normally asymmetrical curves. An example was provided to show how efficient, accurate, and simple the cubic spline method actually was.

In this research, we began by developing a mathematical model using an extension of one-dimensional advection-diffusion equation to simulate seawater intrusion counteracted by flow of freshwater released from an upriver dam, with MacCormack scheme and Cubic Spline interpolation to approximate field data—salinity levels at initial and left boundary conditions.

## II. ONE-DIMENSIONAL SALINITY INTRUSION MODEL

### A. Governing Equation

The mathematical model that we used to describe transportation and diffusion processes of salinity in a river is a one-dimensional advection-diffusion-reaction equation (ADRE) [4, 11, and 12].

$$\frac{\partial S}{\partial t} + U \frac{\partial S}{\partial x} = D \frac{\partial^2 S}{\partial x^2}, \quad \text{for all } 0 \leq x \leq L \text{ and } 0 \leq t \leq T, \quad (2.1)$$

where  $S(x, t)$  is the salinity concentration (g/l);  $U$  is the water flow velocity in the  $x$ -direction (m/s);  $D$  is the salinity diffusion coefficient ( $\text{m}^2/\text{s}$ );  $L$  is the length of lower Chao Phraya river; and  $T$  is the simulated time period.

Letting

$$U = (u_s - ku_w), \quad (2.2)$$

where  $u_s$  is the salinity flow velocity;  $u_w$  is the fresh water flow velocity released by the diversion dam; and  $k$  is the dilution rate of salinity by fresh water, for all  $0 < x \leq L, 0 \leq t \leq T$ .

Substituting Eq. (2.2) into Eq. (2.1), we get

$$\frac{\partial S}{\partial t} + (u_s - ku_w) \frac{\partial S}{\partial x} = D \frac{\partial^2 S}{\partial x^2}. \quad (2.3)$$

### B. Initial and Boundary Conditions

The salinity intrusion in Chao Phraya estuary, which is likely to become more severe in the future, are caused by high ocean tide. In calculation, initial salinity level can be different depending on the time and distance from the estuary.

#### B1. The Initial Condition

The initial condition is assumed to be:

$$S(x, 0) = f(x), \text{ for all } 0 \leq x \leq L, \quad (2.4)$$

where  $f(x)$  is a given salinity level function along the

river.

**B2. The Boundary Condition**

The left boundary condition is assumed to be

$$S(0,t) = g(t), \quad \text{for all } 0 < t \leq T, \quad (2.5)$$

where  $g(t)$  is a given salinity level function at the first monitoring station, closest closed to the estuary. The right boundary condition is assumed to be

$$\frac{\partial S}{\partial x}(L,t) = k_0, \quad \text{for all } 0 < t \leq T, \quad (2.6)$$

where  $k_0$  is the rate of change of salinity level at the last monitoring station, located furthest upriver from the estuary.

**III. NUMERICAL TECHNIQUES**

The solution to the set of equations in Section 2 is numerically approximated by an explicit finite difference method, MacCormack scheme for advection, below.

**A. MacCormack Scheme for Advection**

We can approximate  $S(x_i, t_n)$  by  $S_i^n$ , the value of the difference approximate of  $S(x,t)$  at point  $x = i\Delta x$  and  $t = n\Delta t$ , where  $0 \leq i \leq M$  and  $0 \leq n \leq N$ . The grid point  $(x_n, t_n)$  is defined by  $x_i = i\Delta x$  for all  $i = 0, 1, 2, \dots, M$  and  $t_n = n\Delta t$  for all  $n = 0, 1, 2, \dots, N$  in which  $M$  and  $N$  are positive integers.

**The First Step** of the MacCormack scheme is to approximate equation (2.3) by forward time and forward space scheme (FTFS) as follows:

$$\begin{aligned} S &\cong S_i^n, \\ \frac{\partial S}{\partial t} &\cong \frac{S_i^{n+1} - S_i^n}{\Delta t}, \\ \frac{\partial S}{\partial x} &\cong \frac{S_{i+1}^n - S_i^n}{\Delta x}, \\ \frac{\partial^2 S}{\partial x^2} &\cong \frac{S_{i+1}^n - 2S_i^n + S_{i-1}^n}{(\Delta x)^2}. \end{aligned} \quad (3.1)$$

Substituting (3.1) into (2.3), we get:

$$\left[ \frac{S_i^{n+1} - S_i^n}{\Delta t} \right] + U \left[ \frac{S_{i+1}^n - S_i^n}{\Delta x} \right] = D \left[ \frac{S_{i+1}^n - 2S_i^n + S_{i-1}^n}{(\Delta x)^2} \right] \quad (3.2)$$

for  $1 \leq i \leq M$  and  $0 \leq n \leq N - 1$ . Simplifying (3.2) to:

$$\left[ \frac{S_i^{n+1} - S_i^n}{\Delta t} \right] = -\frac{U}{\Delta x} [S_{i+1}^n - S_i^n] + \frac{D}{(\Delta x)^2} (S_{i+1}^n - 2S_i^n + S_{i-1}^n). \quad (3.3)$$

Define  $(C_i)_1$  by LHS of (3.3), we have:

$$(C_i)_1 = -\frac{U}{\Delta x} (S_{i+1}^n - S_i^n) + \frac{D}{\Delta x^2} (S_{i+1}^n - 2S_i^n + S_{i-1}^n) \quad (3.4)$$

and

$$(C_i)_1 = -\frac{U}{\Delta x} S_{i+1}^n - \frac{U}{\Delta x} S_i^n + \frac{D}{\Delta x^2} S_{i+1}^n - \frac{2D}{\Delta x^2} S_i^n + \frac{D}{\Delta x^2} S_{i-1}^n. \quad (3.5)$$

Let

$$A_1 = -\frac{U}{\Delta x} + \frac{D}{(\Delta x)^2}, A_2 = \frac{U}{\Delta x} - \frac{2D}{(\Delta x)^2}, A_3 = \frac{D}{(\Delta x)^2}.$$

We obtain:

$$A_1 = -\frac{U}{\Delta x} + \frac{D}{(\Delta x)^2}, A_2 = \frac{U}{\Delta x} - \frac{2D}{(\Delta x)^2}, A_3 = \frac{D}{(\Delta x)^2},$$

so we have

$$(C_i)_1 = A_1 S_{i+1}^n + A_2 S_i^n + A_3 S_{i-1}^n. \quad (3.6)$$

For the left boundary, where  $i = 1$ , we obtain

$$\begin{aligned} (C_1)_1 &= A_1 S_2^n + A_2 S_1^n + A_3 S_0^n \\ &= A_1 S_2^n + A_2 S_1^n + A_3 S_0^n. \end{aligned} \quad (3.7)$$

For the right boundary, where  $i = M$ , substitute the approximate unknown value of the right boundary with forward difference approximation to  $\frac{\partial S}{\partial x} = 0$ . Let:

$S_M = S_{M-1}$ , we have:

$$(C_M)_1 = A_1 S_M^n + A_2 S_{M+1}^n + A_3 S_{M-2}^n \quad (3.8)$$

$$= (A_1 + A_2) S_{M-1}^n + A_3 S_{M-2}^n \quad (3.9)$$

we obtain a MacCormack predictor step formulation:

$$S_i^{n+1} = S_i^n + (C_i)_1 \Delta t. \quad (3.10)$$

**The Second Step** of the MacCormack scheme is to approximate equation (2.3) with a backward time and backward space scheme (BTBS) as follows,

$$\frac{\partial S}{\partial t} \cong \frac{S_i^{n+1} - S_i^n}{\Delta t}, \quad \text{condition} \quad \frac{D\Delta t}{(\Delta x)^2} < 0.5, \quad (3.19)$$

$$\frac{\partial S}{\partial x} \cong \frac{S_i^{n+1} - S_{i-1}^{n+1}}{\Delta x}, \quad \text{and} \quad \frac{U\Delta t}{(\Delta x)} < 0.9. \quad (3.20)$$

$$\frac{\partial^2 S}{\partial x^2} \cong \frac{S_{i+1}^{n+1} - 2S_i^{n+1} + S_{i-1}^{n+1}}{(\Delta x)^2}. \quad (3.11)$$

Substitute (3.11) into Eq. 2.3, we get

$$\frac{S_i^{n+1} - S_i^n}{\Delta t} + U \left[ \frac{S_i^{n+1} - S_{i-1}^{n+1}}{\Delta x} \right] = D \left[ \frac{S_{i+1}^{n+1} - 2S_i^{n+1} + S_{i-1}^{n+1}}{(\Delta x)^2} \right]. \quad (3.12)$$

Simplify (3.12) to

$$\frac{S_i^{n+1} - S_i^n}{\Delta t} = -U \left[ \frac{S_i^{n+1} - S_{i-1}^{n+1}}{\Delta x} \right] + D \left[ \frac{S_{i+1}^{n+1} - 2S_i^{n+1} + S_{i-1}^{n+1}}{(\Delta x)^2} \right]. \quad (3.13)$$

Define  $(C_i)_2$  by LHS of (3.13), we have

$$(C_i)_2 = -\frac{U}{\Delta x} (S_i^{n+1} - S_{i-1}^{n+1}) + \frac{D}{(\Delta x)^2} (S_{i+1}^{n+1} - 2S_i^{n+1} + S_{i-1}^{n+1}) \quad (3.14)$$

$$= \frac{D}{(\Delta x)^2} S_{i+1}^{n+1} + \left( -\frac{U}{\Delta x} - \frac{2D}{(\Delta x)^2} \right) S_i^{n+1} + \left( \frac{U}{\Delta x} + \frac{D}{(\Delta x)^2} \right) S_{i-1}^{n+1}.$$

Let

$$B_1 = \frac{D}{(\Delta x)^2}, B_2 = -\frac{U}{\Delta x} - \frac{2D}{(\Delta x)^2}, B_3 = \frac{U}{\Delta x} + \frac{D}{(\Delta x)^2},$$

we have

$$(C_i)_2 = B_1 S_2^{n+1} + B_2 S_1^{n+1} + B_3 S_0^{n+1}, \quad (3.15)$$

for left boundary, where  $i = 1$ .

$$(C_1)_2 = B_1 S_2^{n+1} + B_2 S_1^{n+1} + B_3 S_0^{n+1} \quad (3.16)$$

$$= B_1 S_2^{n+1} + B_2 S_1^{n+1} + B_3 S_0.$$

For right boundary, where  $i = M$ , the unknown values are approximated by backward difference approximation to

$$\frac{\partial S}{\partial x} = 0. \text{ Let } S_{M+1} = S_M, \text{ we have}$$

$$(C_M)_2 = (B_1 + B_2) S_M^{n+1} + B_3 S_{M-1}^{n+1}. \quad (3.17)$$

From the first and second steps, the MacCormack scheme takes on following form:

$$S_i^{M+1} = S_i^n + \frac{\Delta t}{2} \left( (C_i)_1 + (C_i)_2 \right). \quad (3.18)$$

The stable conditions for this scheme are the following,

*B. Interpolation Technique for the Initial and Boundary Conditions*

*B1. Cubic Spline Interpolation*

Salinity measurement data were collected as discrete points in time and space. To construct a continuous model based on those discrete data, some interpolation was necessary. A cubic spline interpolation method could do this interpolation job well. A cubic spline interpolation is a piecewise polynomial approximation, representing the interpolated points in a subinterval with a unique cubic equation. A necessary condition for cubic spline construction is that the first and second derivatives of a cubic spline must be continuous [13, 14].

Suppose  $(x_0, y_0), (x_1, y_1), (x_2, y_2), \dots, (x_n, y_n)$  are  $n + 1$  node pairs where  $a = x_0 < x_1 < \dots < x_n = b$ . Function  $S(x)$  approximating the values between each pair of nodes is called a cubic spline if there exist  $n$  cubic polynomials that satisfy these conditions.

(1)  $S(x)$  is a cubic polynomial on each subinterval  $[x_i, x_{i+1}]$  where  $i = 0, 1, \dots, n - 1$

$$S_i(x) = a_i + b_i(x - x_i) + c_i(x - x_i)^2 + d_i(x - x_i)^3, \quad (3.21)$$

$a_i, b_i, c_i,$  and  $d_i$  are unknown constant coefficients of the cubic polynomial of each subinterval,

$$(2) S_{i+1}(x_{i+1}) = S_i(x_{i+1}) \text{ where } i = 0, 1, \dots, n - 2, \quad (3.22)$$

$$(3) S'_{i+1}(x_{i+1}) = S'_i(x_{i+1}) \text{ where } i = 0, 1, \dots, n - 2, \quad (3.23)$$

$$(4) S''_{i+1}(x_{i+1}) = S''_i(x_{i+1}) \text{ where } i = 0, 1, \dots, n - 2, \quad (3.24)$$

(5) Boundary conditions to be satisfied and interpolated by this cubic spline are the following:

$$(i) S'(x_0) = S'(x_n) = 0 \text{ (natural boundary),} \quad (3.25)$$

$$(ii) S'(x_0) = y'_0 \text{ and } S'(x_n) = y'_n \text{ (clamped boundary).} \quad (3.26)$$

For the natural boundary condition, there are  $n$  linear equations for coefficients  $c_0, c_1, c_2, \dots, c_n$  so we can solve the solution for  $c_i$  from a tridiagonal linear system  $Tx = b$ , where  $T$  is a tridiagonal matrix of  $n \times n$  as follows:

$$T = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 \\ h_0 & 2(h_0+h_1) & h_1 & \ddots & \ddots & 0 \\ 0 & h_1 & 2(h_1+h_2) & h_2 & \ddots & 0 \\ 0 & \ddots & \ddots & \ddots & \ddots & 0 \\ 0 & \ddots & \ddots & h_{n-2} & 2(h_{n-2}+h_{n-1}) & h_{n-1} \\ 0 & 0 & 0 & 0 & 0 & 1 \end{bmatrix}, \quad (3.27)$$

$$b = \begin{bmatrix} \alpha_0 \\ \alpha_1 \\ \vdots \\ \alpha_{n-1} \\ \alpha_n \end{bmatrix} = \begin{bmatrix} 0 \\ \frac{3}{h_1}(a_2-a_1) - \frac{3}{h_0}(a_1-a_0) \\ \vdots \\ \frac{3}{h_{n-1}}(a_n-a_{n-1}) - \frac{3}{h_{n-2}}(a_{n-1}-a_{n-2}) \\ 0 \end{bmatrix}, \quad (3.28)$$

$$\text{and } x = \begin{bmatrix} c_0 \\ c_1 \\ \vdots \\ c_n \end{bmatrix}. \quad (3.29)$$

Then  $b$  and  $x$  are vectors of dimension  $n$ ,  

$$h_i = x_{i+1} - x_i, \quad (3.30)$$

$$\alpha_i = \frac{3}{h_i}(a_{i+1}-a_i) - \frac{3}{h_{i-1}}(a_i-a_{i-1}) \text{ for } i=1,2,\dots,n-1. \quad (3.31)$$

$a_i, b_i$  and  $d_i$  can be calculated by the following equations,

$$a_i = y_i \text{ where } i=0,1,\dots,n, \quad (3.32)$$

$$b_i = \frac{(a_{i+1}-a_i)}{h_i} - \frac{h_i(c_{i+1}-2c_i)}{3} \text{ where } i=0,1,\dots,n-1, \quad (3.33)$$

$$\text{and } d_i = \frac{(c_{i+1}-c_i)}{3h_i} \text{ where } i=0,1,\dots,n-1. \quad (3.34)$$

The cubic function on  $[x_i, x_{i+1}]$ ,  $i=0,1,\dots,n-1$ , with these coefficients can always be written as:

$$S_i(x) = a_i + b_i(x-x_i) + c_i(x-x_i)^2 + d_i(x-x_i)^3. \quad (3.35)$$

#### IV. NUMERICAL EXPERIMENT

##### A. Cubic Spline Interpolation based on Field Data

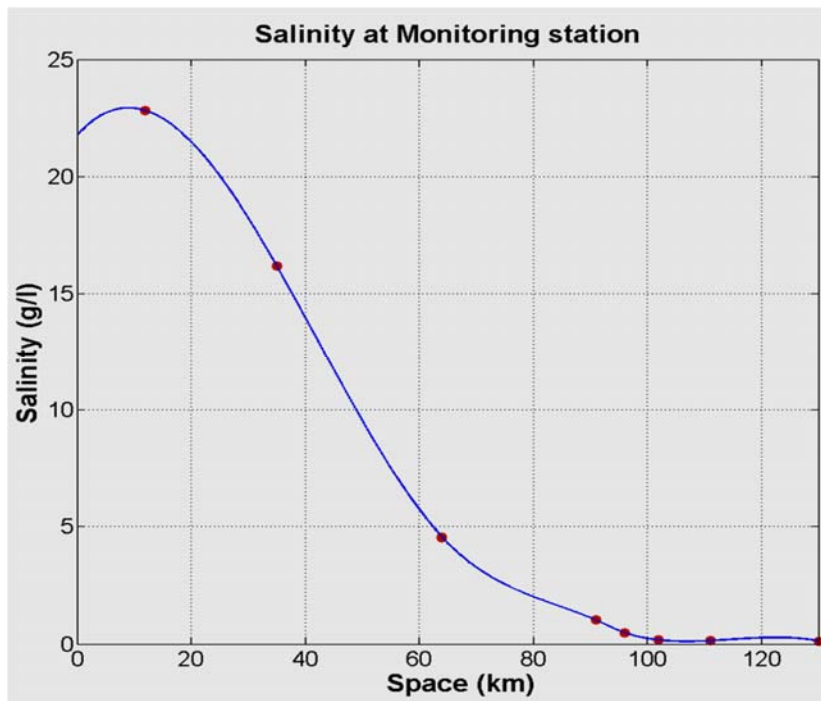


Figure 2. Cubic spline interpolated initial condition  $S(x, 0)$  (g/l)



Figure 3. Cubic spline interpolated left boundary condition  $S(0,t)$  (g/l) (the left boundary was the sea)

*B. Approximation of Salinity Intrusion by MacCormack Scheme*

B1. 4.2.1 Fig. 4–9 are MacCormack generated graphs of salinity level versus distance in space, when  $u_s$  was varied from 0.13, 0.25 and 0.5 m/s and was  $u_w$  varied from 0.52, 0.72, 0.82 and 0.92 m/s, at 4, 8, 12, 16, 20 and 24 hours after the initial time point. The graphs were produced in order to find the optimum value of  $u_s$  and  $u_w$  for the model.

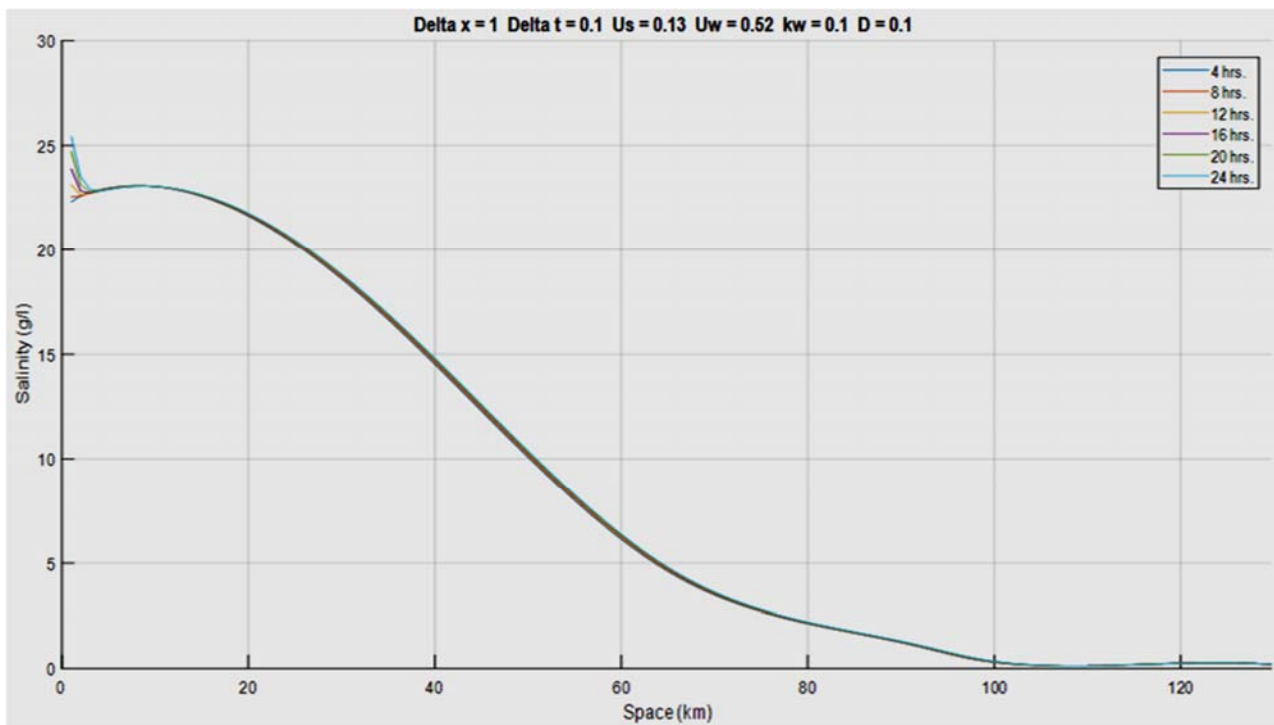


Figure 4. Approximated salinity intrusion levels (g/l) when  $u_s = 0.13$  and at time 4, 8, 12, 16, 20 and 24 hrs after the initial time point

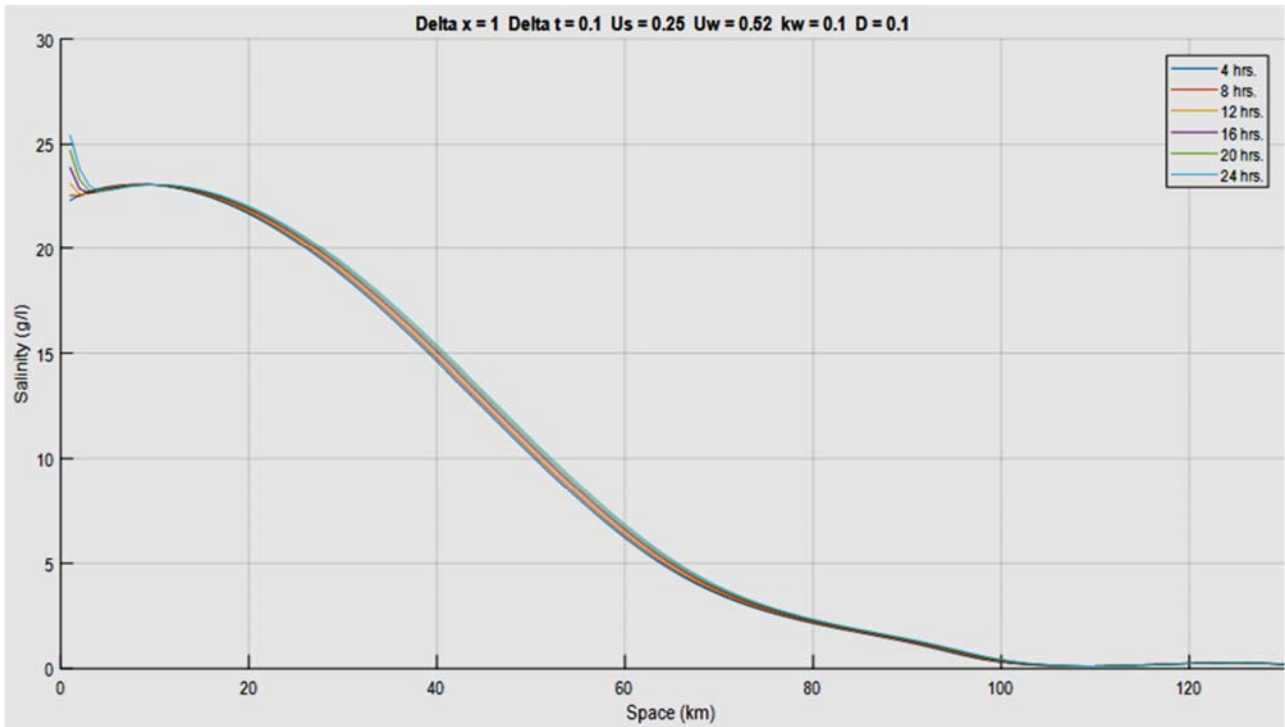


Figure 5. Approximated salinity intrusion levels (g/l) when  $u_s = 0.25$  and at time 4, 8, 12, 16, 20 and 24 hrs after the initial time point

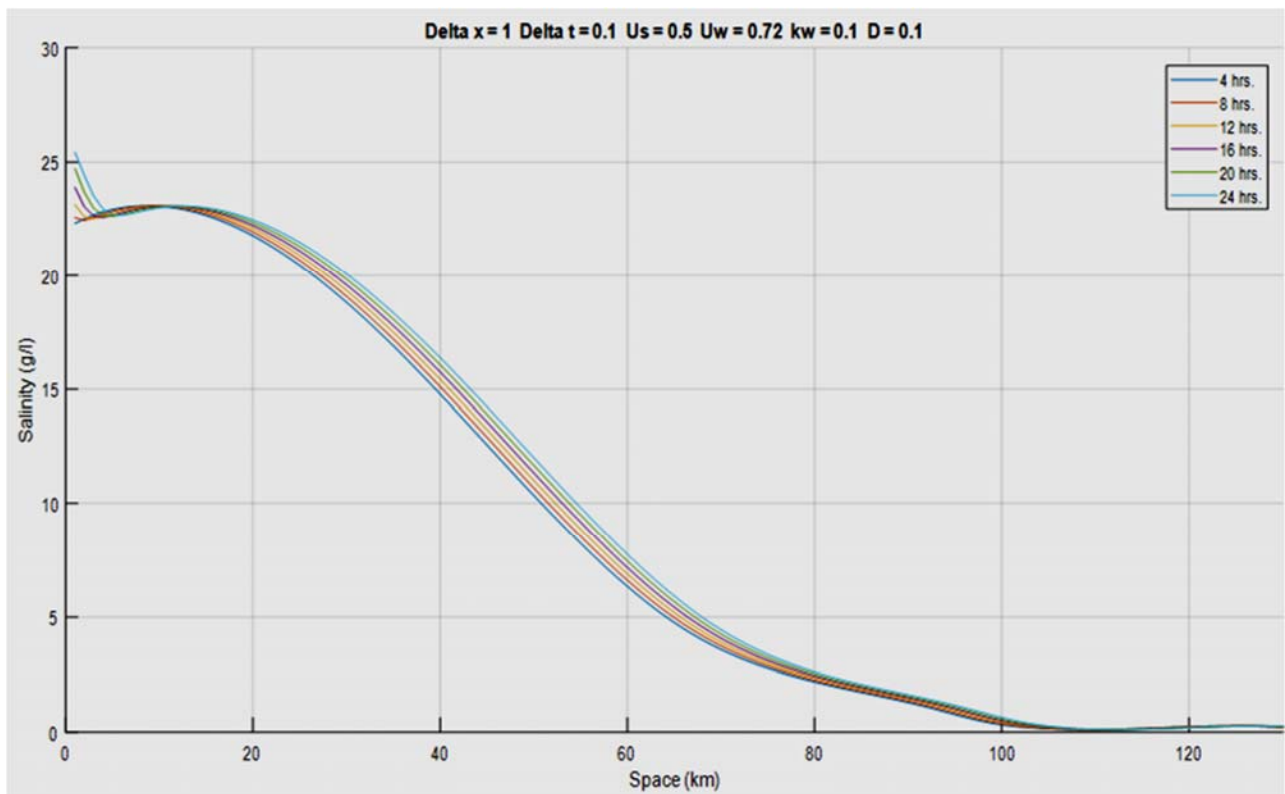


Figure 6. Approximated salinity intrusion levels (g/l) when  $u_s = 0.5$  and at time 4, 8, 12, 16, 20 and 24 hrs after the initial time point

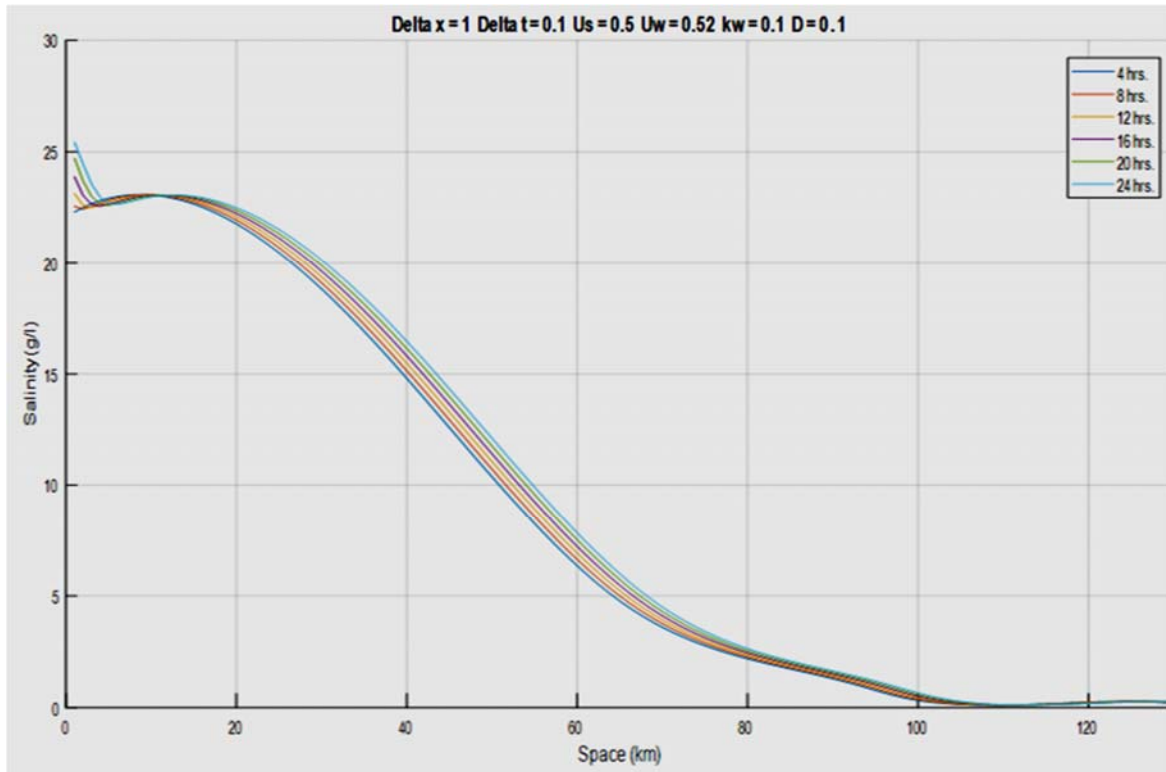


Figure 7. Approximated salinity intrusion levels (g/l) when  $u_s = 0.5$  and at time 4, 8,12,16,20 and 24 hrs after the initial time point

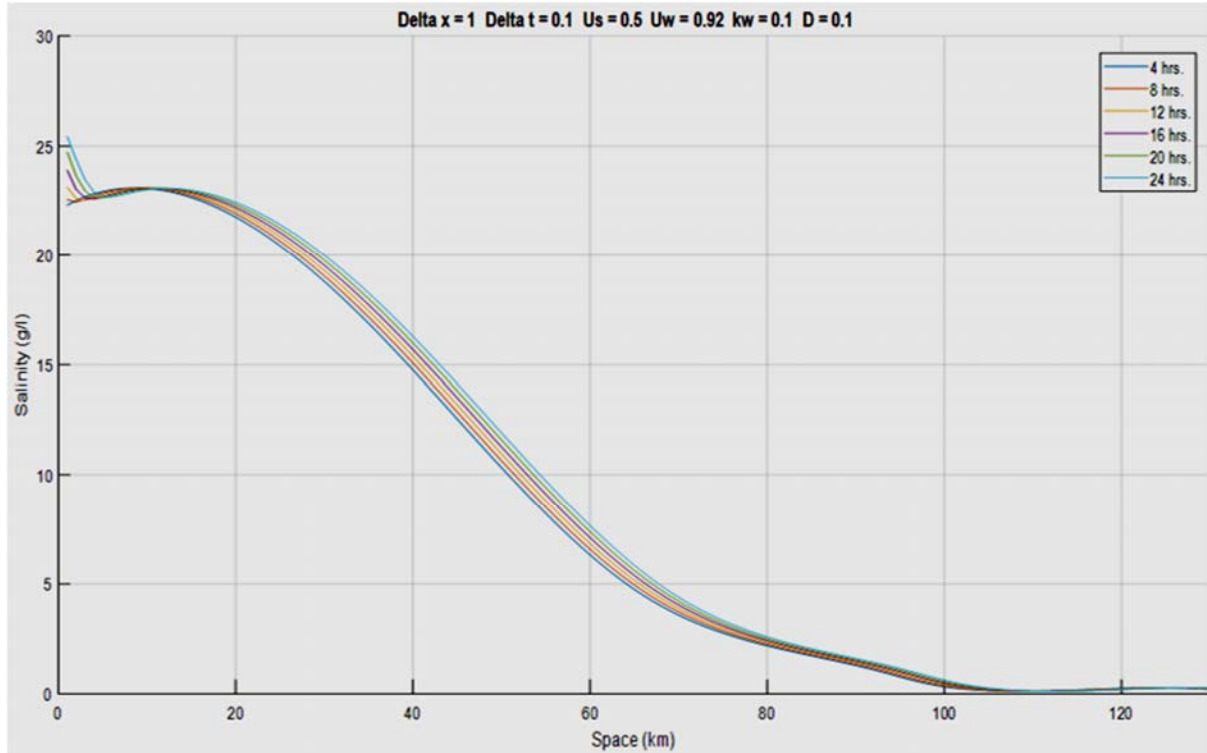


Figure 8. Approximated salinity intrusion levels (g/l) when  $u_s = 0.5$  and at time 4, 8,12,16,20 and 24 hrs after the initial time point.



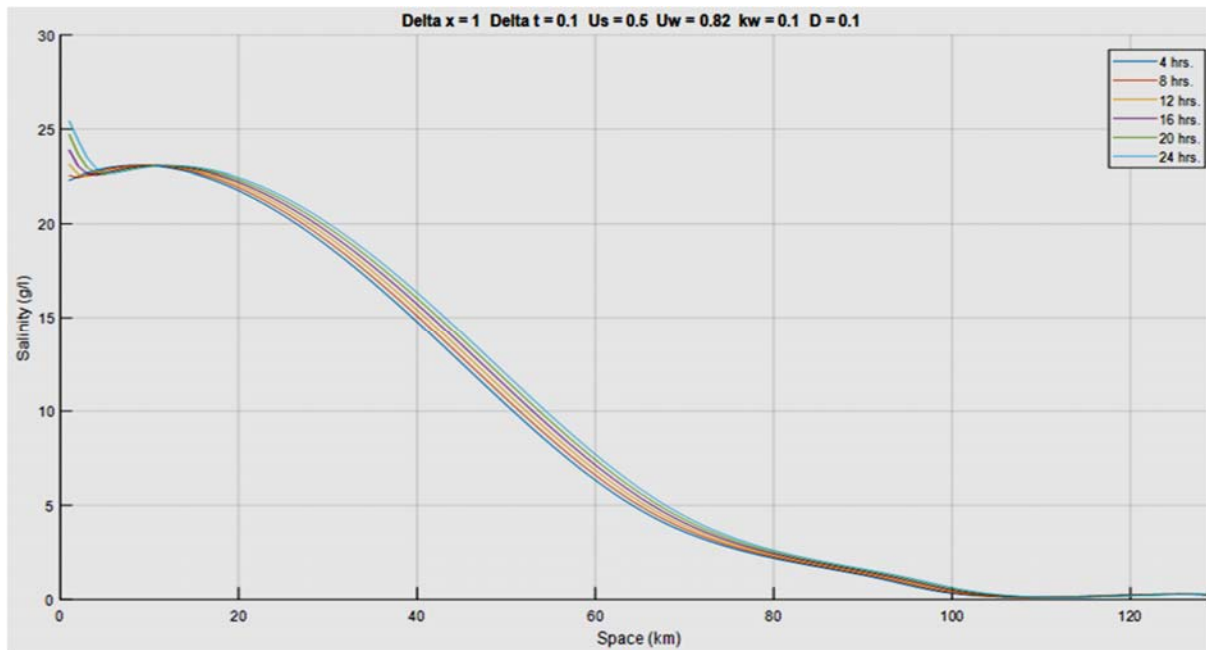


Figure 9. Approximated salinity intrusion levels (g/l) when  $u_s = 0.5$  and at time 4, 8,12,16,20 and 24 hrs after the initial time point

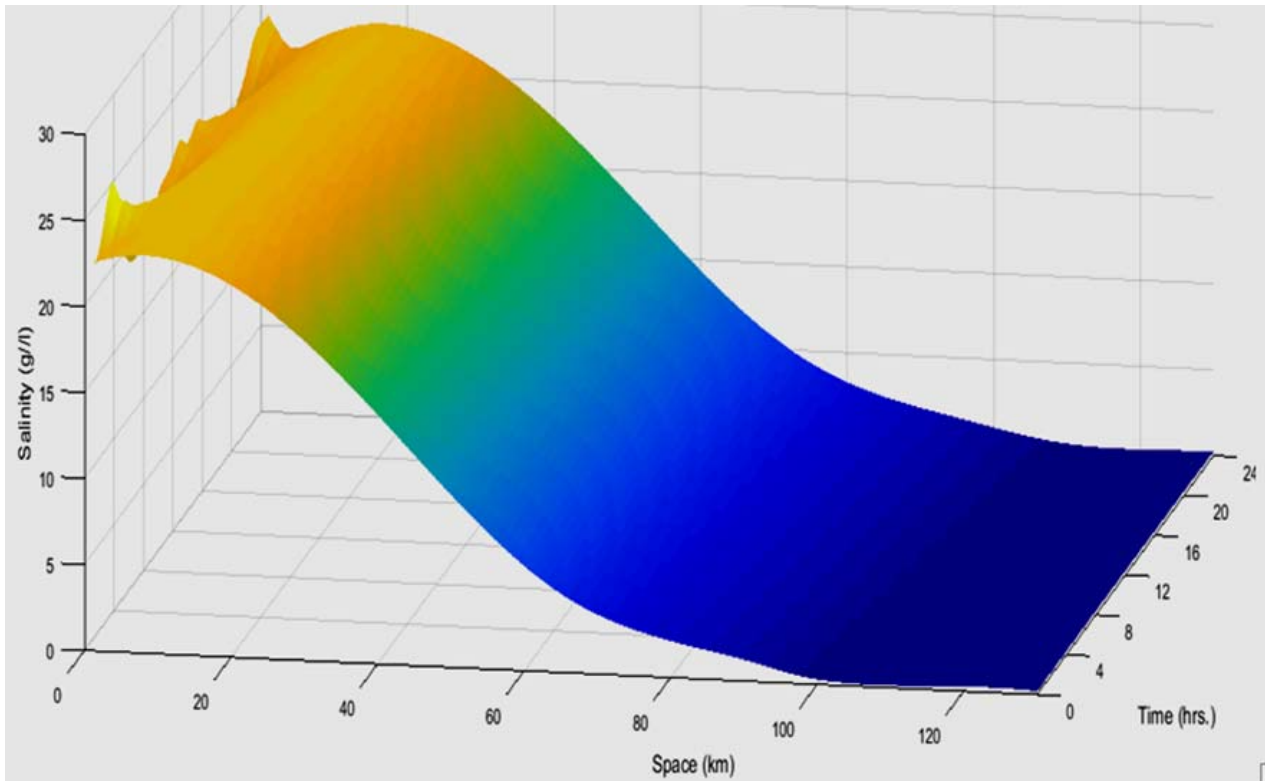
TABLE I APPROXIMATION OF THE SALINITY INTRUSION CONCENTRATIONS (G/L) OF THE MACCORMACK SCHEME SOLUTION WHEN  $u_s$  AS VARIED FROM 0.13, 0.25 AND 0.5 M/S AND  $u_w$  WAS VARIED FROM 0.52, 0.72, 0.82 AND 0.92 M/S,  $D = 0.1$ ,  $k_w = 0.1$  AT DISTANCE 12, 35, 64, 91, 96,102,111 AND 130 (KM) AND AT TIME 4 HRS.

Monitoring Station/ Distance (km)	Salinity Intrusion Concentrations at time 4 hrs. $D = 0.1$ , $k_w = 0.1$						Real data Salinity (g/l) 04-01-2018
	$u_w = 0.52$ , $u_s = 0.13$	$u_w = 0.52$ , $u_s = 0.25$	$u_w = 0.52$ , $u_s = 0.5$	$u_w = 0.72$ , $u_s = 0.5$	$u_w = 0.82$ , $u_s = 0.5$	$u_w = 0.92$ , $u_s = 0.5$	
Phra Nakhon Tai/12	22.8212	22.8431	22.8850	22.8818	22.8802	22.8786	21.84
Khlong Lat Pho/35	16.2369	16.3253	16.5049	16.4908	16.4837	16.4766	12.60
Wat Sai Ma Nuea/64	4.5998	4.6564	4.7743	4.7649	4.7602	4.7555	5.53
Wat Makham/91	1.0339	1.0563	1.1014	1.0979	1.0961	1.0944	0.49
Samlae Pump Station/96	0.5046	0.5237	0.5642	0.5610	0.5593	0.5577	0.25
Wat Phai Lom/102	0.1749	0.1809	0.1940	0.1929	0.1924	0.1918	0.16
Wat Pho Taeng Nuea/111	0.1387	0.1362	0.1314	0.1317	0.1319	0.1321	0.15
Wat Ban Paeng Temple/130	0.1395	0.1442	0.1546	0.1537	0.1533	0.1528	0.13

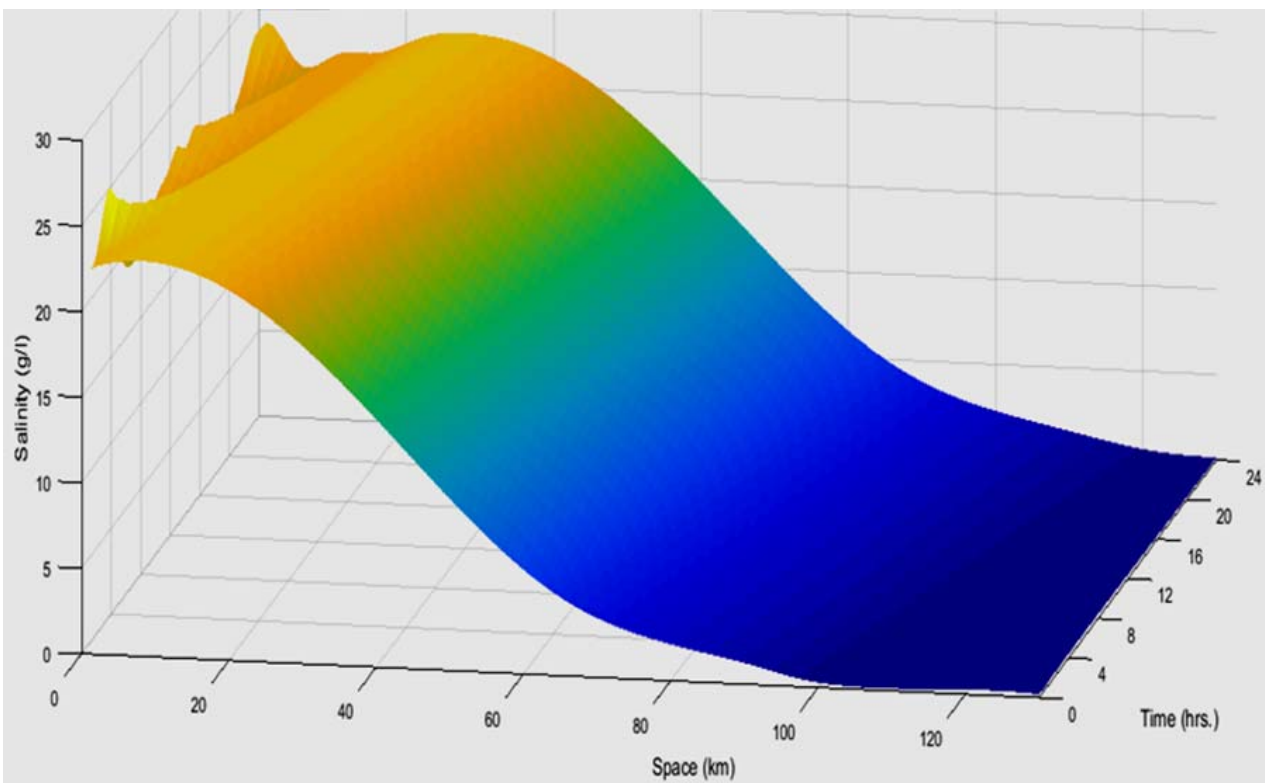
TABLE II. APPROXIMATION OF THE SALINITY INTRUSION CONCENTRATIONS (G/L) OF THE MACCORMACK SCHEME SOLUTION WHEN  $u_s$  AS VARIED FROM 0.13, 0.25 AND 0.5 M/S AND  $u_w$  WAS VARIED FROM 0.52, 0.72, 0.82 AND 0.92 M/S,  $D = 0.1$ ,  $k_w = 0.1$  AT DISTANCE 12, 35, 64, 91, 96,102,111 AND 130 (KM) AND AT TIME 24 HRS.

Monitoring Station/ Distance (km)	Salinity Intrusion Concentrations at time 24 hrs. $D = 0.1$ , $k_w = 0.1$						Real data Salinity (g/l) 04-01-2018
	$u_w = 0.52$ , $u_s = 0.13$	$u_w = 0.52$ , $u_s = 0.25$	$u_w = 0.52$ , $u_s = 0.5$	$u_w = 0.72$ , $u_s = 0.5$	$u_w = 0.82$ , $u_s = 0.5$	$u_w = 0.92$ , $u_s = 0.5$	
Phra Nakhon Tai/12	22.8711	22.9695	23.0632	23.0613	23.0599	23.0583	22.77
Khlong Lat Pho/35	16.5191	17.0368	18.0532	17.9751	17.9358	17.8964	13.55
Wat Sai Ma Nuea/64	4.8035	5.1683	5.9919	5.9233	5.8891	5.8550	4.37
Wat Makham/91	1.1023	1.2292	1.4699	1.4516	1.4424	1.4331	1.27
Samlae Pump Station/96	0.5781	0.7043	0.9829	0.9608	0.9497	0.9386	0.58
Wat Phai Lom/102	0.2026	0.2517	0.3996	0.3854	0.3784	0.3715	0.16
Wat Pho Taeng Nuea/111	0.1330	0.1223	0.1168	0.1163	0.1161	0.1160	0.15
Wat Ban Paeng Temple/130	0.1739	0.2010	0.2511	0.2480	0.2464	0.2447	0.13

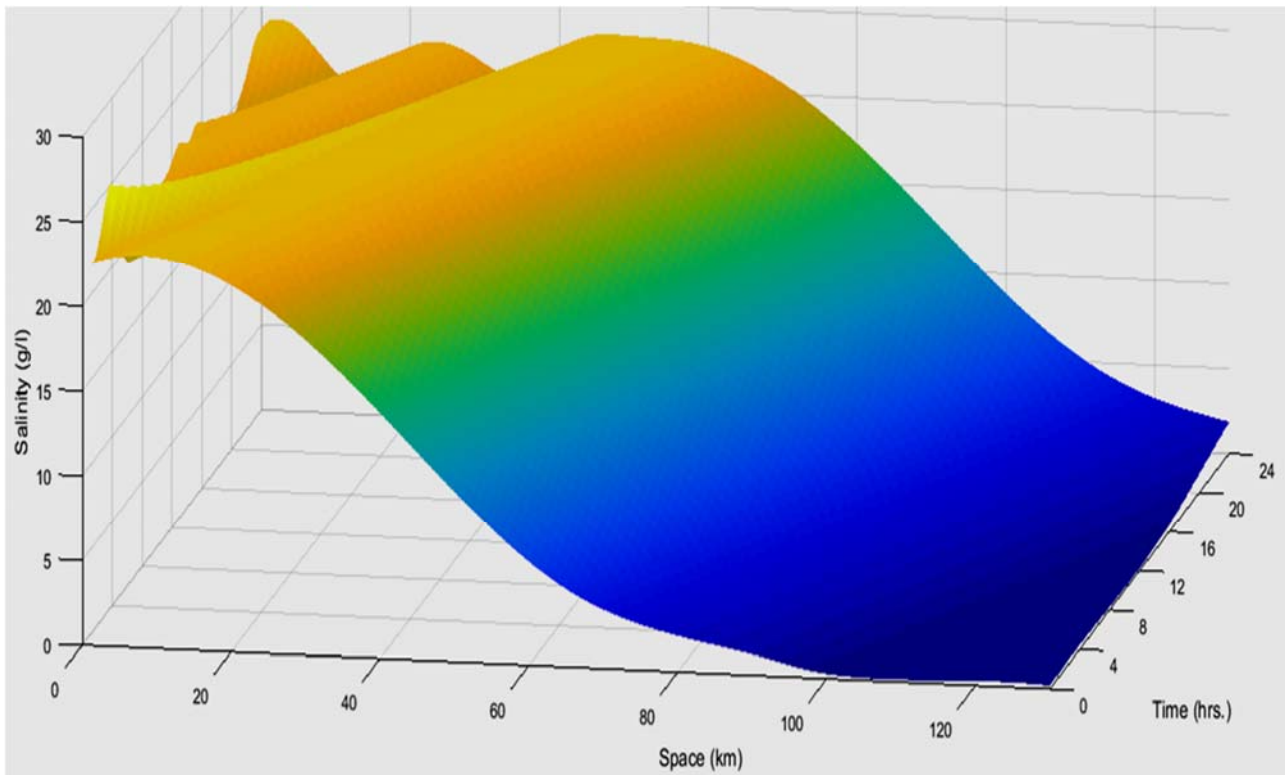
C. The Approximation of the Salinity Intrusion Concentrations of a Simple Advection-Diffusion Equation Numerical Simulation using the MacCormack Scheme.



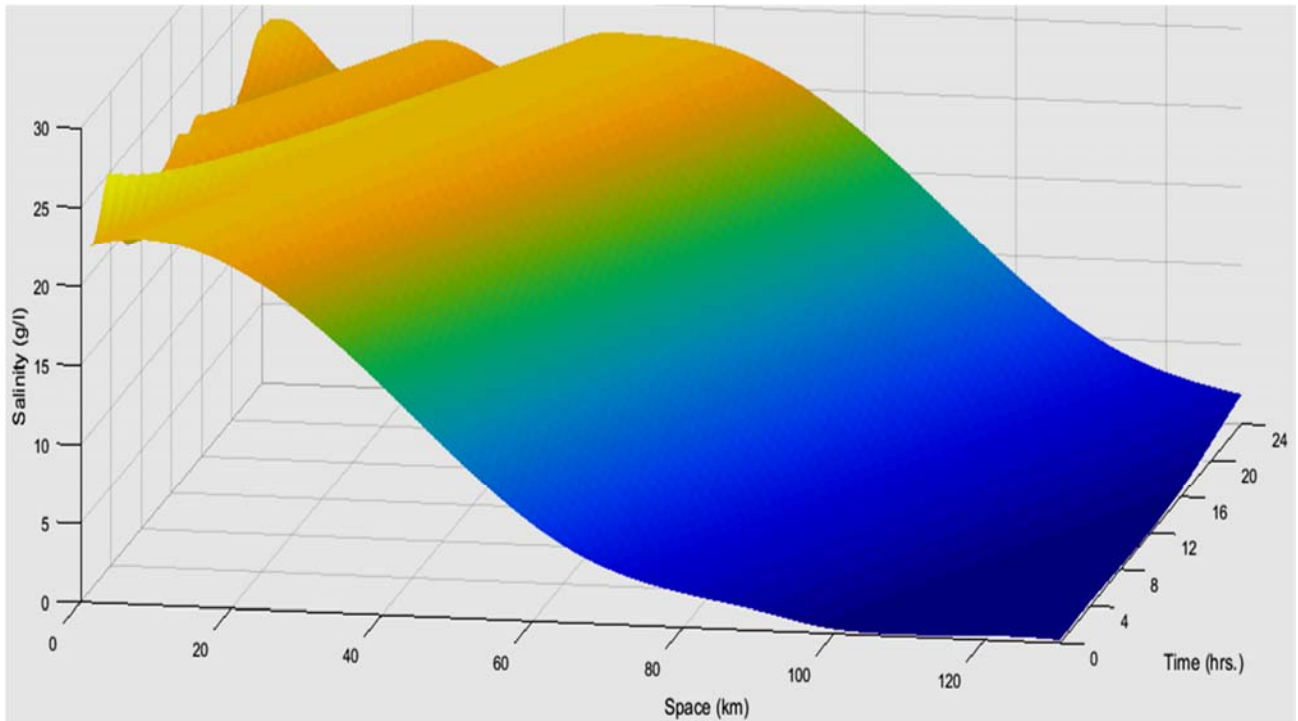
(a): Delta x = 1, Delta t = 0.1, Us = 0.13, Uw = 0.52, kw = 0.1, D = 0.1



(b): Delta x = 1, Delta t = 0.1, Us = 0.25, Uw = 0.52, kw = 0.1, D = 0.1



(c): Delta x = 1, Delta t = 0.1,  $U_s = 0.5$ ,  $U_w = 0.52$ ,  $k_w = 0.1$ ,  $D = 0.1$



(d): Delta x = 1, Delta t = 0.1,  $U_s = 1$ ,  $U_w = 0.52$ ,  $k_w = 0.1$ ,  $D = 0.1$

Figure10. The salinity intrusion concentrations (g/l) of the MacCormack Scheme solution with cubic spline interpolation to the initial and boundary conditions as shown in Surfaces (a), (b), (c), and (d).

## V. DISCUSSION

In this paper, the approximation of the salinity intrusion concentrations of a simple advection-diffusion equation numerical simulation using the MacCormack Scheme is shown in Figures 10. The numerical technique is tested by changing parameters ( $u_w, u_s$ ), which affected the change in salinity intrusion concentrations in six different cases used in this study. We used the same points of time throughout every case, which are at 4, 8, 12, 16, 20 and 24 hrs. respectively. In the first case (Figure 4), the parameters are when  $D = 0.1, k_w = 0.1, u_w = 0.52$  and  $u_s = 0.13$ . In the second case (Figure 5), the parameters are when  $D = 0.1, k_w = 0.1, u_w = 0.52$  and  $u_s = 0.25$ . In the third case (Figure 6), the parameters are when  $D = 0.1, k_w = 0.1, u_w = 0.52$  and  $u_s = 0.5$ . In the fourth case (Figure 7), the parameters are when  $D = 0.1, k_w = 0.1, u_w = 0.72$  and  $u_s = 0.5$ . In the fifth case (Figure 8), the parameters are when  $D = 0.1, k_w = 0.1, u_w = 0.82$  and  $u_s = 0.5$ . In the sixth case (Figure 9), the parameters are when  $D = 0.1, k_w = 0.1, u_w = 0.92$  and  $u_s = 0.5$ , the salinity intrusion concentrations is show in Table 1-2.

## VI. CONCLUSION

The MacCormack Scheme with the cubic spline interpolation to the initial and boundary conditions technique is one-dimensional form of finite difference method. The numerical experiment shows that the calculated results are reasonable approximations. The interpolation technique is suitable to be used to solve the real-world problems because it is easy for computer-coding. According to the field salinity data collected at monitoring stations, the initial and boundary conditions are in the form of functions. The results are very satisfactory because we found that the salinity intrusion concentrations of each monitoring station incline towards the same trend with the actual salinity concentrations measured at each station. The computed results are verified by the numerical accuracy. In summary, the numerical model for salinity intrusion control using the MacCormack finite difference method with Cubic Spline interpolation can be applied with problems in the real-world situations. If there are more parameters, the measured data will be even more accurate. This numerical model is not only easy to analyze, but also save more time and cost to do the field work. Most importantly, the results can be used to support the water management plans in the future.

## CONFLICTS OF INTEREST

The authors declare that there are no conflicts of interest regarding the publication of this paper.

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