Fault Diagnosis of Rotating Machine Based on Audio Signal Recognition System: An Efficient Approach

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***Abstract* - An efficient algorithm for condition monitoring of rotating machines is proposed in this paper. Condition indicators are derived from sound signals, and used to arrive at a decision about the performance state of the machine. Sound signals are recorded by microphones and processed using time-frequency domain analysis. In this study, number of statistical features; such as mean, standard deviation, skewness, and kurtosis are considered. These statistical features were proven to be effective and simple to interpret. Healthy, about to be faulty, and faulty performance states of the machine are considered, and audio signals are recorded for each state. The five main steps comprising the implemented approach are data acquisition, preprocessing, feature extraction, time and frequency domain analysis, and the decision making. Based on the adopted statistical measures, the experimental results indicate that an excellent recognition of machine performance states is obtained, leading to an efficient fault detection and diagnosis.**

Keywords - Singal Processing, Audio Signal; Statistical Analysis; Fault Diagnosis; Vehicle Engines Analysis; Feature Extraction

I. INTRODUCTION

All of us would have the experience that a household equipment such as washing machines or a refrigerator start making unusual sounds. The noisy state is usually accompanied with poor performance and invariably higher power consumption and subsequently a complete stopping of the machines. The noisy state may not last long before the machines stop working. These unusual noises in the machines used in the households or in industry trigger the maintenance procedures. The unusual sounds can be considered as indicators, and may be used to diagnose the conditions of the machines. Based on this, some new technology practices may be applied on various machines. The purpose of these practices is to help in identifying and diagnosing conditions of abnormal operation in machines. Most of the practices rely on monitoring the targeted machines and collecting data about their performance. Sensors can be used to collect data and perform the monitoring task. Collected data is then analyzed to reach proper decisions. It is expected that application of technology for the purpose of monitoring and diagnosing machine anomalies will be more efficient than relying on workers to perform this task. Predictive maintenance is an important strategy that is being adopted in industry in the last few years [1, 2]. This strategy is assumed to improve industrial plant reliability, and minimize unnecessary expenses by avoiding the damage to machines or their parts. In fact, the application of monitoring and diagnosis techniques for the purpose of predictive maintenance has proved, that it enables in increasing the profit, where the cost of maintenance at the proper time is small with respect to the cost of the production loss due to an unscheduled stopping of a machine with little or no notice.

The interest of introducing fault detection, tolerance and redundancy in a system is to increase its safety and reliability [3], by adopting proper methodologies and trying to predict fault occurrence before it actually takes place. One way to do this is to employ Fault Detection and Diagnosis systems (FDD) [4, 5]. Powerful techniques that are normally included with the fault detection and diagnosis systems comprise of calculations and the use of advanced skills in signal processing.

Signal processing is often used in many fields for feature extraction and classification. Examples of such fields are; the medical diagnosis [6, 7], industrial process control [8], and numerous other fields. The reason behind using the signal processing techniques in diverse applications is to provide necessary information that assist in decision making [9]. Signal processing techniques can be useful in fault detection of automobile rotating motors [16, 17]. Signal processing techniques are normally classified into three main categories, namely, 'time', 'frequency' or 'time-frequency' domains-based algorithms respectively. Fig. 1 illustrates the typical approaches and their features used in extraction and classification of audio signals [10].

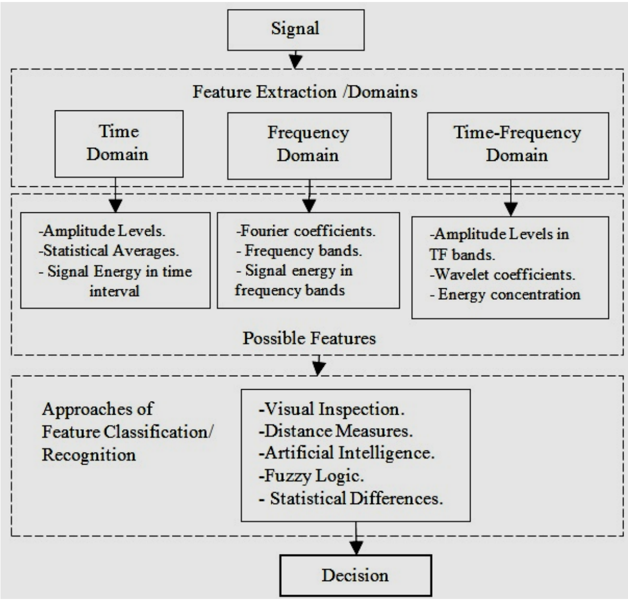

Figure1. Signal processing for pattern classification in a typical application.

The time-frequency analysis has proven to be as a vital technique, when signal models are not available. In such cases, it is not possible to provide comprehensive information, if either 'time' or 'frequency' domain is used. When the time-frequency transform is adopted, it is ideally possible to get direct information about the frequency components occurring at any given time. This is achieved by combining the local information of a frequency spectrum with the temporal behavior of a given signal [11].

Internal Combustion Engines (ICE), which are widely used in cars, initiate their running by the aid of starter motors. If a fault occurs within the starter motor, the ICE cannot be run. Therefore, application of fault diagnosis for car engines is very important. In [12], Bay and Bayir observe the values of currents and voltages related to a starter motor, and propose a fault diagnosis technique for the motor based on fuzzy logic methodology. In our paper, we adopt a similar machine and utilize the time-frequency domain analysis of a set of audio files. The sound data is recorded for a starter motor at different performance states. Number of statistical parameters are computed and used as indicators for deciding on the performance state of the starter motor.

The paper is organized as follows: section 2 introduces literature review, in section 3, the methodology used in our study is illustrated. Section 4 presents the results and discussions. Conclusion is presented in section 5.

II. RELATED WORKS

Among the several techniques used for anomaly detection and fault diagnosis in rotating machines, sound analysis is an effective technique of condition monitoring. This technique relies on the analysis of sounds that are recorded for the monitored machine at different performance states. It is not easy to collect many sound files for such machines for the purpose of detecting their abnormal performance. Despite of this difficulty, many researchers have taken a keen interest in this technique as it leads to meaningful and reliable results.

In [13], Uematsu et. al, stated that the abnormal performance of an equipment can be determined by measuring the deviation from a recoded sound of normal state, where a predefined threshold is used to take this decision. Other researchers have considered some acoustic features from the analyzed sound signals, where in [14], Wan and Chen considered fault detection in a rolling element bearing, and proposed a method that is based on the kurtosis wave and information divergence for fault. They presented a practical example to illustrate the application of their proposed method. Related techniques used for fault diagnosis and condition monitoring of roller element bearing are introduced in [15]. Aref, et. al. [16, 17] exploit the spectrum analysis to detect the fault in automobile rotating machine. It was presented there, that the rotating machine sound signals produces distinct patterns in different performance conditions of the machine.

Halim et. al. [18] considered the time domain averaging and wavelet transform to extract the periodic waveform of the noisy signal from the vibration signals of gears. The technique's main emphasis is to filter the noise and detect the faults in the monitored gears. Gear fault diagnosis based on Kurtosis criterion is also studied by Xiao et. el. [19], where the instantaneous frequency mean is calculated as the evaluation index, and the characteristic curve is drawn to screen out the most relevant Intrinsic Mode Functions (IMFs) of the original vibration signal. Then, the kurtosis value feature vectors of IMFs are normalized and input into Self-Organizing Map (SOM) neural network to realize gear fault diagnosis. Wang et. al. [20] proposed a composite fault diagnosis method for gearboxes based on an improved Ensemble Local Mean Decomposition (ELMD). Their idea is to add white noise in pairs to optimize ELMD, then remove the decomposed high noise component using appropriate filter.

Further fault classification algorithms for gears and other rotating machines are proposed in literature adopting various signal processing techniques. In [21], Support Vector Machine (SVM) is considered, while Artificial Neural Network (ANN) is proposed in [22]. Wavelet transform and deep learning are adopted by the work of Heydarzadeh et. al. [23]. Gearbox fault diagnosis models are proposed in [24] based on genetic algorithm, while wavelet support vector machine is considered in the paper of Chen, whereas the fault diagnosis technique proposed in [25] was based on Hilbert-Hung transform. In [26], however, Cocconelli et. al. focused on the computation of scalar quantities used for condition monitoring. They worked on vibration data collected from an AC motor running at a constant speed with different types of imposed faults.

A global parameter to make better characterization of fault condition is proposed in [27], where integration of the difference between the power spectrum density of fault condition and the normal one is used. Analysis of collected sound data from railway structures are used in [28] to detect and diagnose faults, whereas Mel-Frequency Cepstrum Coefficients (MFCCs) are extracted and employed with Support Vector Machines (SVMs). In [29], an SVM based solution is proposed for fault detection in wind turbines. A decision tree-based online condition monitoring is proposed in [30] for diagnosing faults in gears. In this study, fuzzy and neuro controller is trained using number of statistical parameters being extracted from time domain vibration signals.

III. METHODOLOGY

The methodology of this research is divided into two parts; namely the dataset collection, and the description of the proposed approach.

A. Dataset

Three categories of data sets, which are sound files recorded for a 12V DC starter motor. The categories are related to different performance conditions of the adopted machine. Non faulty or Healthy Machine (HM), About to be Faulty Machine (AFM), and Faulty Machine (FM) are the three considered conditions. Ten audio recordings, representing the data samples with 3-5 seconds per sample are made for each condition. An appropriate audio recorder with a sensitive microphone is used for recording these audio samples.

B. The Proposed Approach

The proposed approach consists of the following six main steps, as shown Fig. 2:

- **Data acquisition:** In this step, the sound of the vehicle engine is recorded using sensitive sensors. The recorded audio files are of different durations, and the sampling rate applied is 22050 Hz.

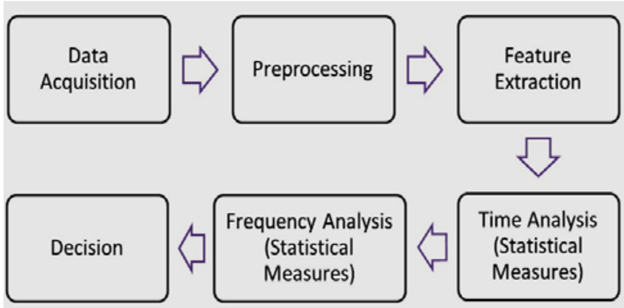

Figure 2. Steps of the proposed approach.

- **Preprocessing:** In this step, the sound signal passed through a resizing process in order to achieve unification in the signal sizes. The output of this process generates files with 60000 samples.
- **Moving Average Filter:** The unified size sound files that are generated from the preprocessing step, are passed through a one-dimensional Moving Average filter (MA filter) in order to remove the unwanted noise. The MA filter is a commonly used tool to smooth an array of sampled data and remove the unwanted noise. This filter is similar to a Low Pass filter, and it is used for smoothing of sampled signals [31]. L-point discrete moving average filter can be represented as below:

$$y(n) = \frac{1}{L}\sum^{L-1} x(n-k) \qquad (1)$$

where x represents the input and y represents the average output.

- **Feature Extraction:** In this step, an efficient set of features are generated in order to recognize and diagnoses the fault.
- **Statistical Measures:** Some selected statistical measures are considered for application. The choice, and hence application of these statistical measures produces values that are utilized in the process of decision making. Among the well-known statistical measures are; the mean value to observe the central tendency of data representation, the dispersion of data is indicated by variance or standard deviation values, the symmetry of data is shown by skewness value, and finally the kurtosis value is used to illustrate the shape of data representation. These statistical measures are implemented on both time domain and frequency domain signals.

To explain the required statistical measures, consider x as the digital input audio signal having N samples, where $k= 1, 2, 3, \ldots, N$. The mean value, which is related to the central tendency of the signal, is given by the following equation [32]:

$$\mu = \sum_{k=1}^{N} \frac{x_k}{N} \qquad (2)$$

The equation for calculating the variance value that is showing the dispersion of data is as follows:

$$\sigma^2 = \sum_{k=1}^{N} \frac{(x_k - \mu)^2}{N} \tag{3}$$

The symmetry of data is measured by the skewness value and it is given by the following equation [33]:

$$Skewness = \frac{1}{N}\sum_{k=1}^{N}\left[\frac{(x_k - \mu)}{\sigma}\right]^3 \tag{4}$$

Kurtosis value indicating the shape of data, is given by the following equation [34]:

$$Kurtosis = \frac{1}{N}\sum_{k=1}^{N}\left[\frac{(x_k - \mu)}{\sigma}\right]^4 \tag{5}$$

- **Decision:** In this step, a proper strategy is applied in order to take the appropriate decision by analyzing values from the calculated statistical measures. Two main factors are considered to recognize a faulty state:
  - A classifier that compares the current value of the condition indicator to values associated with fault states.
  - A threshold value that indicates a fault when the indicator exceeds it.

IV. RESULTS AND DISCUSSIONS

Time-frequency domain results are recorded and analyzed. Number of diagrams are produced illustrating the feature extraction and statistical measure production. Three states of machine operation are considered and introduced in our proposed approach for fault detection. These states are Healthy Machine (HM), about to be Faulty Machine (AFM), and Faulty Machine (FM). In Figs. 3-5, the spectral frequency analysis and time frequency analysis for the three operational states of the machine (i.e. Healthy Machine (HM), About to be Faulty Machine (AFM), and Faulty Machine (FM) are presented respectively.

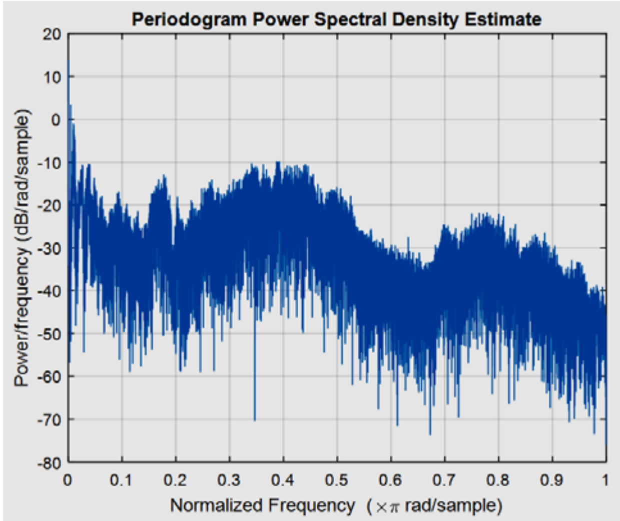

Figure 3a. Spectral frequency of healthy machine (HM).

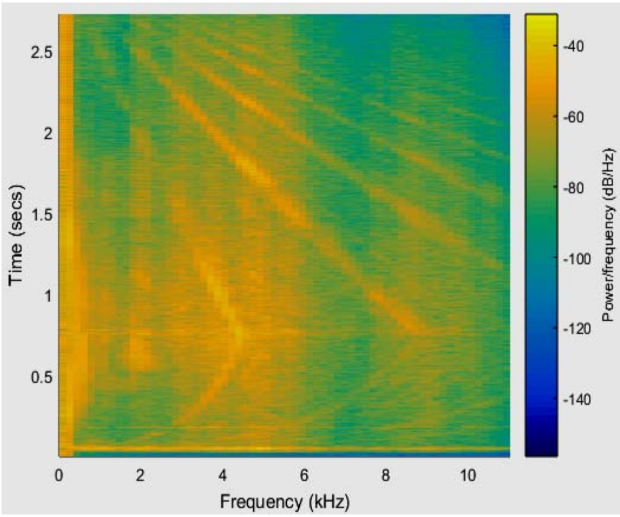

Figure 3b. Time frequency of healthy machine (HM)

In Fig. 3a and Fig. 3b the spectral frequency, and time frequency analysis for Healthy Machine (HM) state of operation are shown. It is clear from the time frequency analysis diagrams that the situation of signal spectrum lines is having some uniformity that can be easily recognized. The clear appearance of spectrum lines in this state is due to the fact that noise signal is low and is basically being smoothed.

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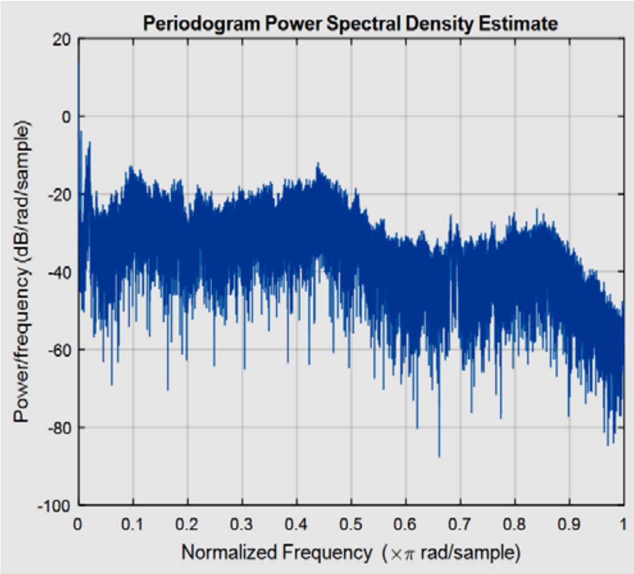

Figure 4a. Spectral frequency of about to faulty machine (AFM).

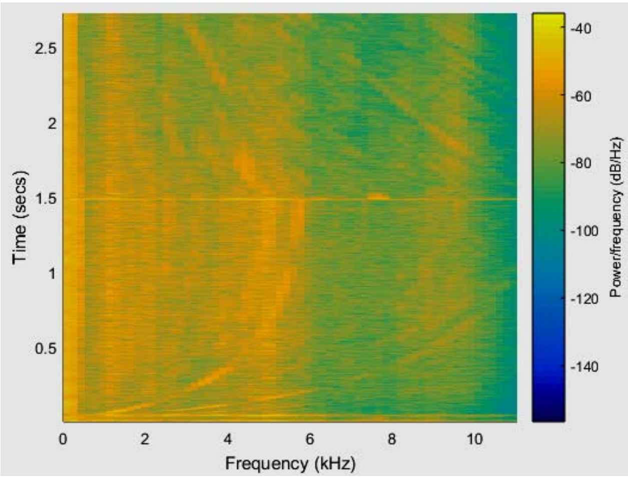

Figure 4b. Time frequency of about to faulty machine (AFM).

Fig. 4a and Fig. 4b illustrates the spectral frequency analysis and time frequency analysis for the About to be Faulty Machine (AFM) state of operation. In this figure, the spectrum indicates a noticeable difference compared to that of the previous figure. The noise signal in this figure is increased, which can be noticed from the loss of uniformity affecting part of the spectrum. This difference between the two spectrums is simply related to the fact that they belong to two slightly different audio signals.

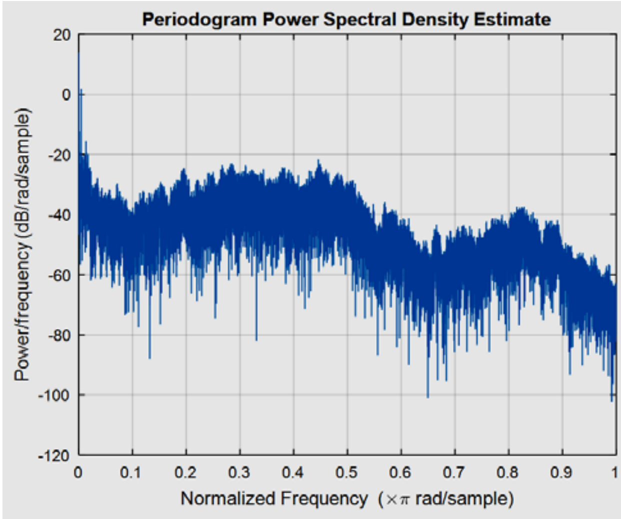

Figure 5a. Spectral frequency of faulty machine (FM).

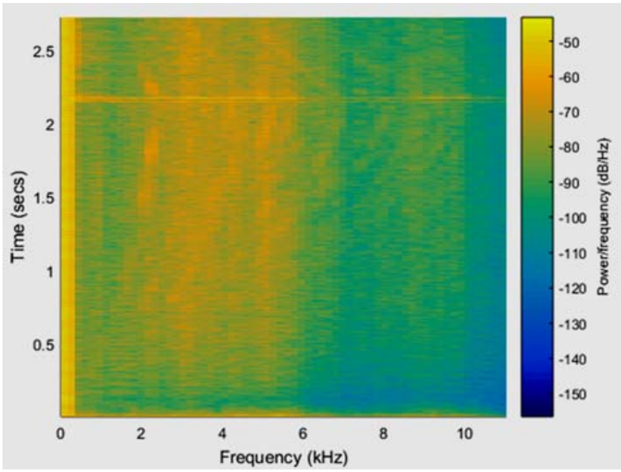

Figure 5b. Time frequency of faulty machine (FM).

Fig. 5a and Fig. 5b shows the spectral frequency analysis and time frequency analysis. In this figure, it is apparent that the spectrum is scattered and no more uniformity is shown in the diagram, where the noise signal has increased and hence the machine is producing a different sound. This recorded sound represents an audio signal constituting large amount of noise compared to the audio signal recorded for a healthy machine in its normal operation.

The following part of the results and calculations focuses on time domain statistical measures of the aforementioned three operational states of the machine. Four of the well-known statistical measures; namely mean, standard deviation, skewness and kurtosis, are considered. The proper formulas are applied to find out values of these measures and the obtained results are illustrated in graphs in Figs. 6-9.

In Fig. 6, the measurement of the mean value according to time domain signal is shown for all the three considered operational states of the machine. The mean value of the healthy state signal has some fluctuations at the first four values. Such fluctuations may be related to the starting part of the audio signal, before it becomes more stable. The mean value of the signal related to the (about to be faulty) machine is also appearing in Fig. 6, and it shows smooth fluctuations at the mean values of the audio signal. The mean value of faulty machine signal has smooth fluctuations at the mean values of the audio signal, in addition the average of all sound signals is -0.0507, as shown in Table I.

TABLE I. AVERAGE OF THE TIME DOMAIN STATISTICAL MEASURES

Measure	Operational States of the machine		
	Healthy	About to be faulty	Faulty
Mean	-0.0512	-0.0513	-0.0507
Standard deviation	0.1913	0.0832	0.0296
Skewness	0.0764	0.0766	0.12766
Kurtosis	4.833	13.932	40.736

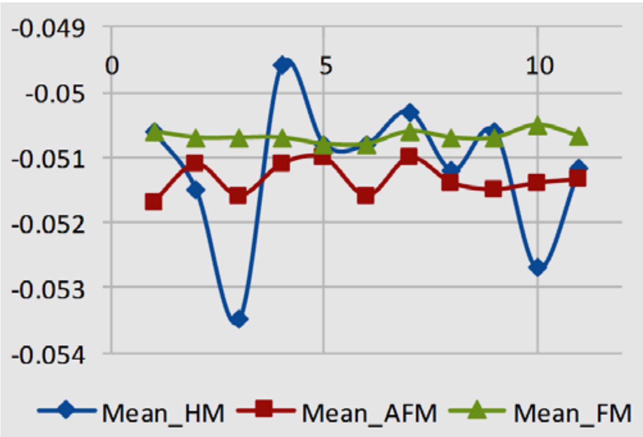

Figure 6. Mean values in time domain.

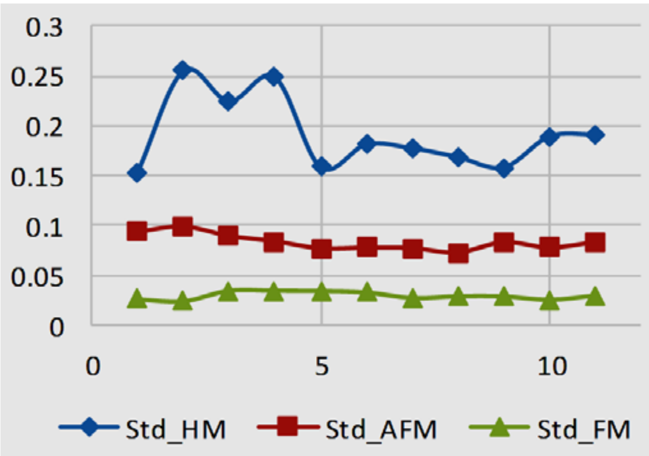

Figure 7. Standard deviation values in time domain.

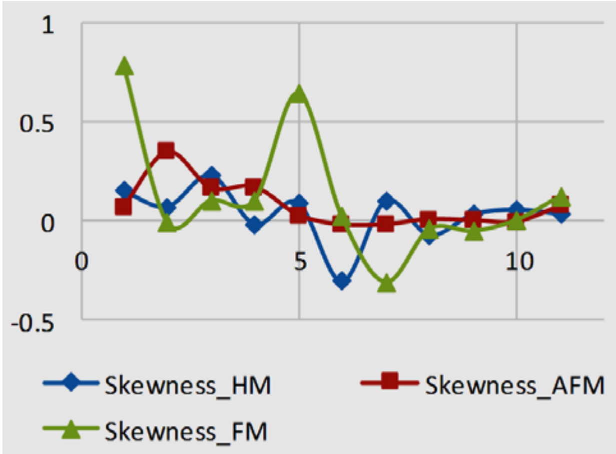

Figure 8. Skewness values in time domain.

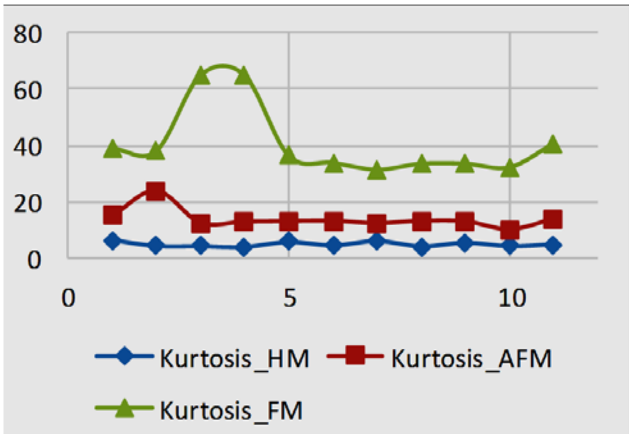

Figure 9. Kurtosis values in time domain.

Figures 10-13 concentrate on the frequency domain statistical measures of the three states. These statistical measures are mean, standard deviation, skewness and kurtosis values.

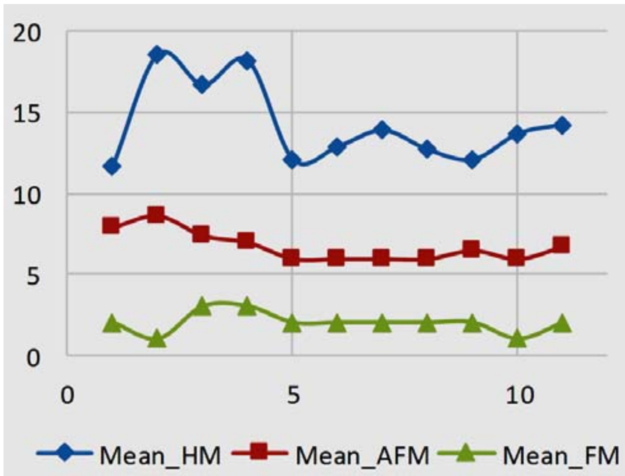

Figure 10. Mean values in frequency domain.

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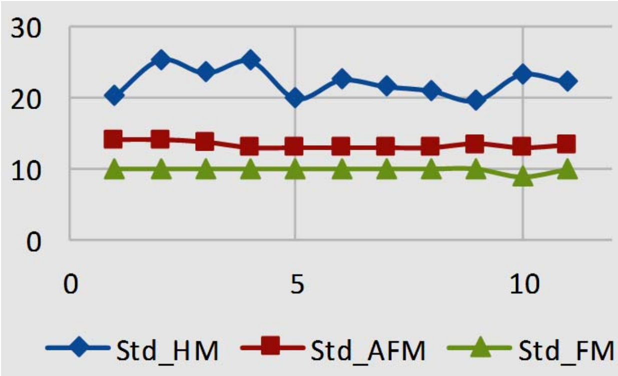

Figure 11. Standard deviation values in frequency domain.

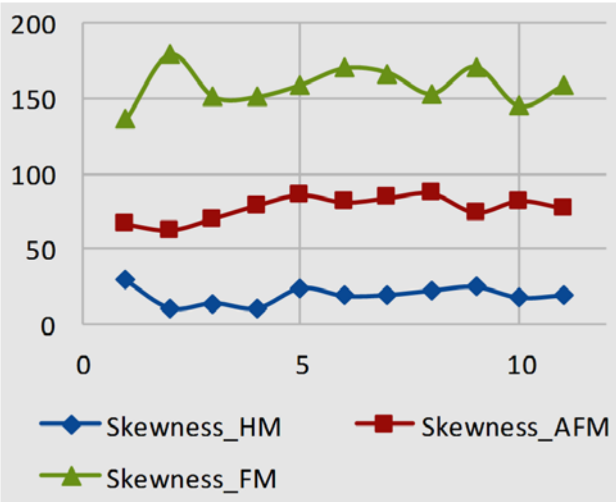

Figure 12. Skewness values in frequency domain.

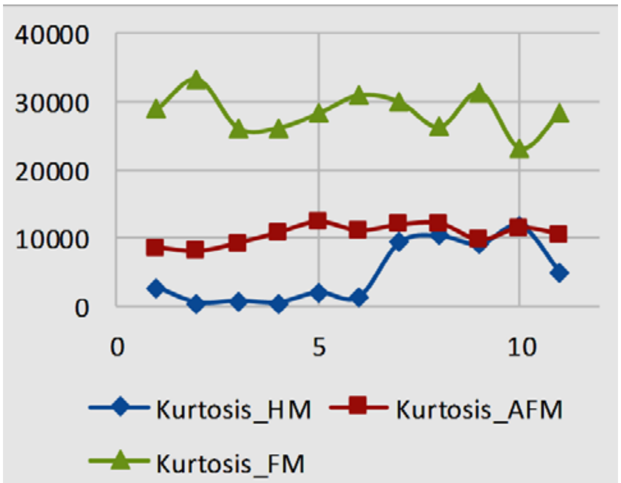

Figure 13. Kurtosis values in frequency domain.

Table II shows the average values of the frequency domain statistical measures for healthy, about to be faulty and faulty machines respectively. This table indicates a significant difference between healthy, about to be faulty and faulty machines. So, it is straight forward to detect the state of the machine by analyzing their sound's statistical measures. A correlation function is applied to compare these values with that of stored values.

TABLE II. AVERAGE VALUES OF THE FREQUENCY DOMAIN STATISTICAL MEASURES

Measure	Operational States of the machine		
	Healthy	About to be faulty	Faulty
Mean	14.26	6.74	2
Standard deviation	22.33	13.32	9.9
Skewness	18.8	77.1	158.2
kurtosis	4769.42	10536.05	28445.7

V. CONCLUSIONS

An important issue of the machine life cycle is to know the onset of the fault of that machine. In addition, the correct decision at a right time is very important to identify the machine fault via certain analysis procedures. Three machine states: healthy machine, about to be faulty and faulty machine are tested via the proposed system. This research guides both time domain and frequency domain analysis. In time domain analysis we obtained an effective recognition in both standard deviation and kurtosis in which the state of the machine can be directly identified. On the other hand, the general view of the obtained results indicates that all the performed statistical measures implemented in frequency domain contribute significantly in the differentiation between the three states which are healthy machine, about to be faulty and faulty machine. The obtained results indicated a high recognition rate of the machine state.

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