Trapezoidal Based Fuzzy Membership Functions for Student Model Design

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Abstract - **Designing the input/output membership function is the first major step in the three steps processes of a fuzzy logic controller. The process involves determining the most suitable fuzzy logic algorithm capable of transforming the crisp data entering the fuzzy controller and assigning appropriate fuzzy terms and their corresponding degree of membership in the categories. The shape of a membership function graph can take various representations such as trapezoidal, triangular, bell curves, singleton or any form that accurately enables the distribution of information within the system, in as much as the shape provides a region of transition between adjacent membership functions. The main goal of this paper is to propose the design of fuzzy membership functions that represent the distribution and prediction of student's knowledge performance in an Intelligent Tutoring System (ITS).**

Keywords - Fuzzy Logic Controller, Fuzzy Membership Function, Student Model, ITS

I. INTRODUCTION

Decision making processes in most real world environments are largely affected by high degree of imprecision and "uncertainties" which accounts for the lack of the exact knowledge of the consequences, goals and constraints of possible actions. These "imprecision" and "uncertainties" are the central points of fuzzy set theories and applications. The technique of fuzzy logic was invented to address most of those issues in real world environments that are characterized by the inherent high level of "imprecision" and "uncertainties" [1]. And since its discovery, the technique of fuzzy logic has proven to be a successful tool in most Artificial Intelligent (AI) and engineering domains ranging from intelligent decision support, automobile applications as well as mass market consumer products.

Fuzzy logic technique primarily deals with reasoning that is approximate rather than fixed and exact. In other words, the concept of fuzzy logic enables reasoning and making rational decisions in circumstances of imprecision, uncertainty, human subjectivity, incomplete information and deficient computations [3]. The basic element of the fuzzy logic theory is the fuzzy set. A fuzzy set describes a characteristic, thing, fact or state. For example, "novice" is a fuzzy set that describes knowledge level, "young" is a fuzzy set that describes age, "cold" is a fuzzy set that describes a body temperature, "tall" is a fuzzy set that describes height, "loud" is a fuzzy set that describes sound's intensity, "close" is a fuzzy set that describes the distance between two objects [2].

The student model, is a quantitative representation of the actual student's characteristics and a vital component of an ITS. It is created based on the observations and predictions the System make on each student. The student's knowledge is one of the most dynamic characteristic of a learner; so dynamic like a moving target. However, modeling student's knowledge and diagnosis are complex processes that are characterized by uncertainty and imprecision [17]. One approach for dealing with these issues is fuzzy logic technique which has similar way of expressing the natural human conceptualization through linguistic variables which are the core of membership functions. These membership functions are tools for simulating fuzziness in human cognition and they are subjective and context-dependent which makes their choice a key problem in the design of a fuzzy logic controller [18].

II. RELATED LITERATURE

A. Overview of Membership Functions

Fuzzy logic systems are rule-based systems that use the theory of fuzzy sets and fuzzy logic introduced by Zadeh [1] to encounter imprecision and uncertainty. It deals with reasoning that is approximate rather than fixed and exact. It is a precise logic of imprecision and approximate reasoning. In other words, fuzzy logic is able to reason and make rational decisions in circumstances of imprecision, uncertainty, human subjectivity, incomplete information and deficient computations [9]. Many applications are using fuzzy logic systems to represent knowledge in a closer way to how human are thinking. In a fuzzy set, any element in the set is given a degree of membership of this set as opposed to the ordinary crisp set where its membership is characterized by two values only (0 or 1). A general fuzzy logic system involves fuzzifying crisp values followed by inference engine to apply fuzzy rules and ends by defuzzifying the results into crisp values as outputs [4]. The ability of fuzzy logic to handle the uncertainty, imprecise and incomplete data, and information that is characterized by human subjectivity makes it useful in many human centric fields [10]. Fuzzy set theory has been applied in the design and development of intelligent tutoring systems, more specifically in student model design.

A fuzzy set A of a universe of discourse X (the range over which the variables span) is characterized by a membership function of the form:

$$\mu_A(x) : X \rightarrow [0;1] \quad (1)$$

Equation (1) associates with each element x of X a number $\mu_A(x)$ in the interval [0,1], with $\mu_A(x)$ representing the degree of membership of x in A [5]. The membership function, often given the symbol μ, is the basis of fuzzy set theories. It is a curve that defines how each point in the input space (the crisp input) is mapped to a fuzzy term and a corresponding degree of membership usually taken as a real number in the interval [0,1]. The crisp inputs are mapped into the membership functions on the antecedent part defined by fuzzy rules to obtain the corresponding fuzzy terms or linguistic variables and a corresponding degree of membership for each term. The use of membership functions in the fuzzification processes has created more alternatives to assign membership values to fuzzy terms than there are to assign probability density values to random variables [6]. Membership functions are subjective and context-dependent that means that it is hard for a system to automatically generate them in a concrete and formal way. The choice of membership functions is a key problem in the design of a fuzzy controller and therefore the way to select them is usually determined by experts, based on its suitability in terms of simplicity, convenience, speed, and efficiency [7]. The membership functions can take one of the symmetric or asymmetric forms of triangles, exponential Gauss, trapezoid and so on in the general unified form. The peculiarity of a fuzzy set is that objects belong to the set with a certain degree, called membership value. This value is a continuous number ranging from 0 (complete exclusion) to 1 (complete membership) [8].

B. Architecture of a Fuzzy Logic System

The general architecture of a Fuzzy logic system comprises of three fundamental components namely, fuzzifier, Inference engine and defuzzifier [11].

From figure 1, it can be seen that an input to the Controller is usually in the form of Crisp (numerical) values that the membership function will read and transform each crisp input to its corresponding linguistic variable and of membership for inference mechanism. To transform the output back to its crisp form a defuzzifier is employed and inputs can then be stored in the knowledge container.

C. The Student Model Component

Intelligent Tutoring Systems (ITSs) are special classes of E-learning systems designed to provide adaptive and personalized tutoring based on the individuality of students [12]. An important component of an ITS that is responsible for providing the basis for this personalization is called the student model. During the course of interaction between the learner and an ITS, a quantitative representation of the actual student's characteristics such as knowledge state and learning style is created based on the observations and predictions the ITS made on each student. The student's knowledge is one of the most dynamic characteristic; so dynamic like a moving target. However, modeling student's knowledge and diagnosis are complex processes that are characterized by uncertainty and imprecision issues that affect the prediction of the student model in an ITS. Fuzzy logic is a form of multi-valued logic derived from fuzzy set theory. An approximate student model based on fuzzy membership function approach enables making accurate predictions about the state of student's knowledge.

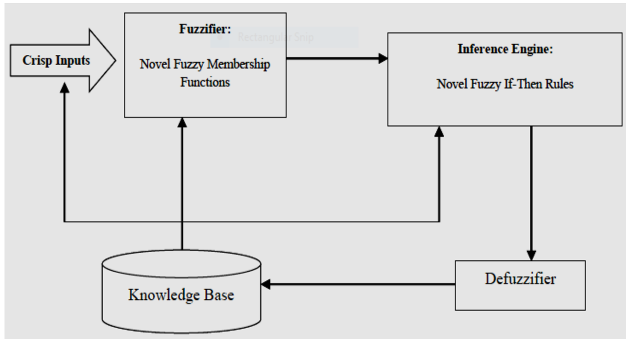

Figure 1. Architecture of a Fuzzy Logic System

Student modeling is a vital process in ITS where the tutoring system observes students' actions and creates quantitative representations of each learner's properties of interest, which are vital to other ITS modules. The main goal of a student model is to support making instructional decisions based on the individuality of the learners. A good student model that matches student behaviors to student properties of interest can often provide insightful information to both the system and the researchers. Student behaviors can be viewed as the input of a student model, which include a variety of observations, such as student responses and actions. Student's characteristics of interest represent what the student is being modeled.

Depending on the requirements, the range of things being modeled could be fairly broad: student knowledge, student performance, student emotions, learning styles and other constructs of interest. Student models create quantitative representations, which are consumable to other modules within the ITS, and most of which are also interpretable to humans outside the system. It has been said that a well-designed tutoring system actively undertakes two tasks: that of the diagnostician, discovering the nature and extent of the student's behavioral attitudes such as knowledge, and that of the strategist planning, a response using its findings about the learner. This is the main role of

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student model, which is the base for personalization in intelligent tutoring systems [14].

The information of a student model is used by the system in order to adapt its responses to each individual student dynamically providing personalized instruction, help and feedback. The student model is used for accurate diagnosis in order to predict students' needs and adapt the learning materials and processes to each individual student's learning pace. Moreover, by predicting of student affective state, an adaptive and/or personalized educational system can select appropriate learning methods in order to increase the effectiveness of tutorial interactions and improve the learning and motivation. In addition, a student model can be used for identifying the student's strength and weaknesses in order to provide him/her with customized or individualized feedback [15].

Designing the proposed input/output membership functions is the first step of a fuzzy logic control process which is very vital in creating the proposed fuzzy student model. The input/output membership functions are primarily designed to transform the crisp inputs, in this case the student's knowledge test scores entering the system by assigning fuzzy terms and corresponding membership degrees to the variables in those categories. However, it is known that there are no defined criteria for designing or selecting which shape the membership function can take, instead, it is a matter of judgment of the expert designer to choose from either triangular, trapezoidal, bell curves or any other shape as long as those shapes accurately represent the distribution of information within the system, and as long as a region of transition exists between adjacent membership functions [13].

III. METHODOLOGY

This section provides the description of part of the training data and criteria for the design of all three membership functions that are vital in modeling the students' knowledge performance in a domain concept. Fuzzifier-A, Fuzzifier-B and Fuzzifier-C as well as the corresponding linguistic variables and the co-ordinates in each function are also explained.

TABLE I. PART OF THE TRAINING DATA

Domain Concept	Knowledge Test Score
Arithmetic Operation	0.375
Arithmetic-logic Unit	0.375
Central Unit	0.5
Central Unit	0.25
Fortran	0.125
Diskette	0.25
DOS	0

A. Membership Function Graphs for Fuzzifier-A

The graph of the Fuzzifier A consist of six membership functions representing the six linguistic variables poor, weak, average, Good, V.Good and Excellent. The co-ordinates of the points in each membership function in the Fuzzifier is defined (Table II).

TABLE II. CO-ORDINATES FOR FUZZIFIER_A

Fuzzy Set	Tuned Co-ordinates
Poor	[(0, 0),(0,1),(0.05,1),(0.1,0)]
Weak	[(0.05, 0),(0.15,1),(0.25,1),(0.3,0)]
Average	[(0.25,0),(0.35,1),(0.45,1),(0.5,0)]
Good	[(0.45,0),(0.55,1),(0.65,1),(0.7,0)]
V. Good	[(0.65,0),(0.75,1),(0.85,1),(0.9,0)]
Excellent	[(0.85,0),(0.95,1),(1,1)]

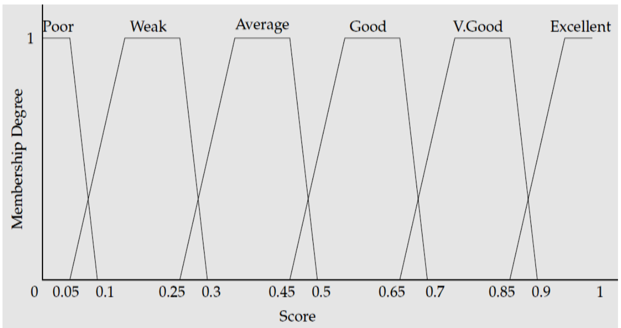

Figure 2. Graph of Fuzzifier A

B. Membership Function Graphs for Fuzzifier-B

The graph of the Fuzzifier-B consist of six membership functions representing the six linguistic variables poor, weak, average, Good, V.Good and Excellent. The co-ordinates of the points in each membership function in the Fuzzifier is defined (Table III).

TABLE III. CO-ORDINATES FOR FUZZIFIER-B

Fuzzy Set	Tuned Co-ordinates
Poor	[(0,0),(0,1),(0.05,1),(0.15,0)]
Weak	[(0.05,0),(0.15,1),(0.25,1),(0.35,0)]
Average	[(0.25,0),(0.35,1),(0.45,1),(0.55,0)]
Good	[(0.45,0),(0.55,1),(0.65,1),(0.75,0)]
V. Good	[(0.65,0),(0.75,1),(0.85,1),(0.95,0)]
Excellent	[(0.85,0),(0.95,1),(1,1)]

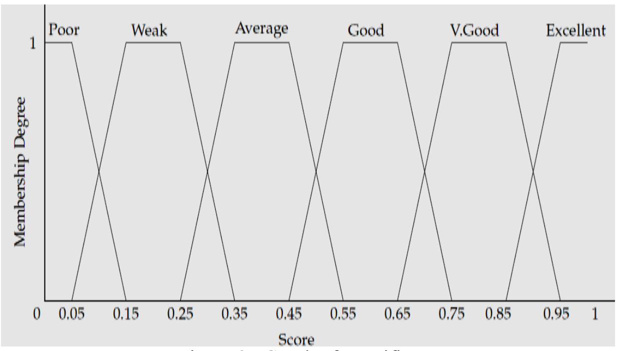

Figure 3. Graph of Fuzzifier-B

C. Membership Function Graphs for Fuzzifier-C

The graph of the Fuzzifier-C consist of six membership functions representing the six linguistic variables poor, weak, average, Good, V.Good and Excellent. The co-ordinates of the points in each membership function in the Fuzzifier is defined (Table IV).

TABLE IV. CO-ORDINATES FOR FUZZIFIER-C

Fuzzy Set	Tuned Co-ordinates
Poor	[(0,0),(0,1),(0.05,1),(0.1,0)]
Weak	[(0.08,0),(0.15,1),(0.25,1),(0.3,0)]
Average	[(0.28,0),(0.35,1),(0.45,1),(0.5,0)]
Good	[(0.48,0),(0.55,1),(0.65,1),(0.7,0)]
V. Good	[(0.68,0),(0.75,1),(0.85,1),(0.9,0)]
Excellent	[(0.88,0),(0.95,1),(1,1)]

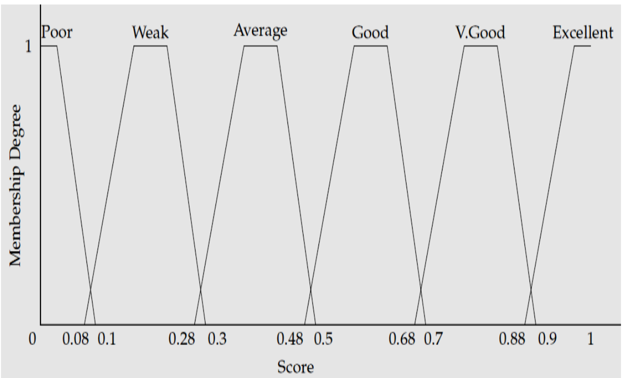

Figure 3. Graph of Fuzzifier-C

IV. RESULTS AND DISCUSSION

The study uses a training data which is an instance of a knowledge test administered to thirty students from two South-East European Universities, University of Split, Croatia and University Bosnia-Herzegovina. The knowledge test consist of seventy two concepts from the domain of Computer Science (TABLE I). The performance for each student on a domain concept was recorded within the interval (0,1) and is used as the crisp inputs to the Fuzzy Controller.

Based on the literature discussed in the review section, the choice of trapezoidal fuzzy membership function as inference based approach to designing the fuzzy student model is to due to its suitability in mapping the multi-linguistic variables used in predicting student's knowledge performance for accurate representation and distribution of information within the system.

A. Criteria for Defining the Membership Functions

Each membership function is defined by four points with each point having distinct coordinates (x_i,y_i) on a plane. The x_i's represents student's score in a domain concept from the universe of discourse (TABLE I) and the y_i's represents a horizontal intervals for membership degrees [0,1]. Because of its flexibility and linearity, this study propose the design of trapezoidal-based membership functions for the six linguistic variables "poor", "weak", "average", "good", "very good" and "excellent" to enable the realization of the first fuzzy control process, the fuzzification process. The proposed trapezoid membership functions are specified by four points with each point being represented in the x-y plane by its co-ordinates (x_i,y_i). The four vertices and their corresponding arcs that define the graph of each fuzzy set must produce a trapezoidal shape with flat top, the core of the membership function when plotted on the x-y plane. A region of transition also known as intersection region between two adjacent membership functions must exist. The membership functions for the three fuzzifiers must be distinct in either core, transition (intersection) region or in their lower or upper bounds. With these criteria, this study has successfully tuned the membership function's coordinates that will enable the design of three novel fuzzifiers, Fuzzifier-A, Fuzzifier-B and Fuzzifier-C.

V. CONCLUSION

This study has successfully designed three novel fuzzy based membership functions control process which model student's knowledge performances as in fuzzifier-A, fuzzifier-B and fuzzifier-C using six linguistic variables Poor, Weak, Average, Good, V. Good and Excellent. Successful design of these membership functions, the first and most vital components of any fuzzy logic controller has enables the realization of three fuzzy student models. An approximate student model based on fuzzy membership function approach enables making accurate predictions about the state of student's knowledge which will in turn allow for necessary diagnosis on them. The main goal of designing the three models is to enable successful realization of the next fuzzy logic control process, the fuzzy inference process to allow for the necessary diagnosis on the fuzzy student models.

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