Chaotic Attractor Generated by Combining Chua Attractor with Another Circuit

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Abstract - This study presents new simulations of chaotic behavior by combining the Chua circuit with other circuits. The Chua circuit serves as an ideal subject for exploring complex dynamics. Through meticulous simulations, we scrutinize the circuit's response under various input conditions, unveiling complex patterns and behaviors characteristic of chaos. Our results contribute to a deeper understanding of the chaotic properties inherent in electronic systems, potentially paving the way for applications in various fields. Additionally, integrating the Chua circuit with other components and treating it as a Sierpinski triangle in simulation studies provides valuable insights into the captivating realm of chaos. Furthermore, our observations include many cases of convergence and stability in chaotic systems.

Keywords - Chua circuit, chaotic, chaotic attractor, oscillator, resistive splitter, small signal, triangle Sierpenski.

I. INTRODUCTION

The Chua attractor is a mathematical model that exhibits chaotic behavior, and it is commonly implemented using the Chua circuit, an electronic circuit named after its creator, Leon O. Chua. The Chua circuit is known for its ability to generate chaotic signals [17, 19].

Exploring cascade configurations with the Chua circuit offers a fascinating opportunity to analyze the limits and possibilities of this nonlinear electronic system. In this pursuit, our approach involves integrating specific elements, such as a resistive splitter arrangement and a small-signal differential circuit and Sierpinski triangle, into the Chua circuit. This endeavor aims to reveal the complex interactions and emerging behaviors resulting from the combination of these components, thereby paving the way for a comprehensive understanding of the dynamics of the cascade circuit.

II. CHUA CIRCUIT

The Chua circuit is an example of a nonlinear dynamic system that can exhibit chaotic behaviors. It is a third-order dissipative electrical circuit, thoroughly investigated both experimentally and numerically. It utilizes the Chua diode as a key nonlinear component. The circuit must include:

- One or more nonlinear elements.
- One or more locally active resistances.
- Three or more energy storage elements.

In 1983, engineer Leon Ong Chua developed the simplest electronic circuit meeting these criteria. It consists of two capacitors, a coil, an active resistor, and a Chua diode.

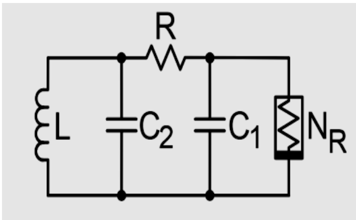


Figure1. The Chua circuit.

The Chua diode is a nonlinear electronic component with the interesting property of generating chaotic behaviors in a circuit, and it is the most essential part of the Chua circuit. The dynamics of the circuit are defined by the following equations:

$$\begin{cases} C_1\dot{V}_1 = \dfrac{V_2 - V_1}{R} - f(V_1) \\ C_2\dot{V}_2 = \dfrac{V_1 - V_2}{R} + i \\ L\dot{i} = -V_2 \end{cases} \qquad (1)$$

With:

- V□ and V□, representing the voltage
- Capacitors C_1 and C_2
- i denoting the current flowing through the inductance, and f representing the current response of the Chua diode.

The Chua diode has been replaced by a cascade arrangement based on operational amplifiers (TL082) and resistors R to replicate the functioning of the Chua diode.

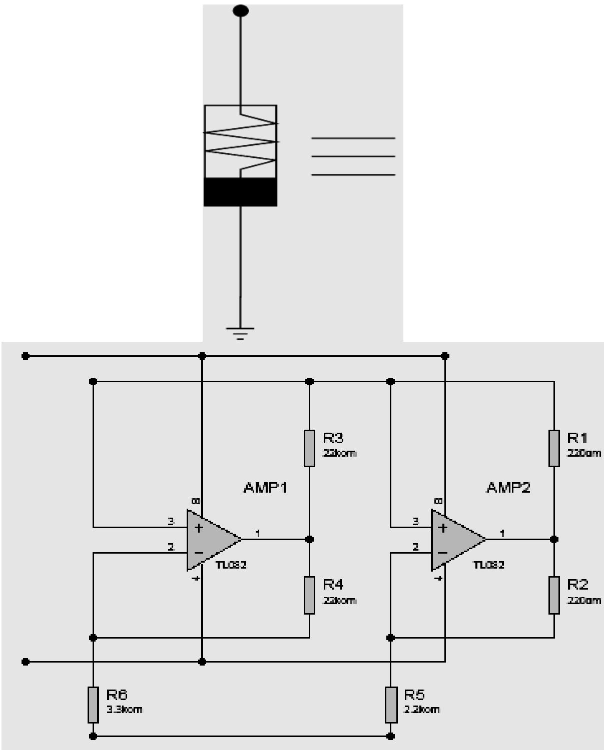


Figure 2. A circuit that replaces the Chua diode.

We will attempt to simulate a Chua circuit (using ISIS Proteus) and later explore how to synchronize two or three circuits for present chaotic attractor.

Figure 3 shows an assembling of the Chua circuit with cascade amplifier.

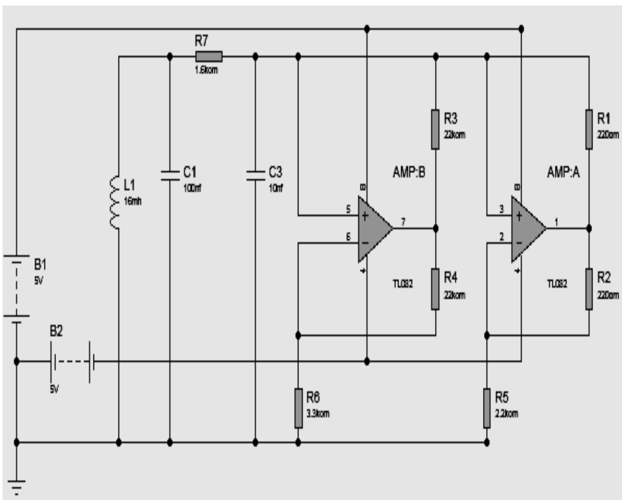


Figure 3. The Chua circuit with cascade amplifier.

III. SIMULATIONS OF CHUA ATTRACTORS

A. Chua Attractor with Oscillator:

Circuit 1: Differential Small-Signal Circuit

A "small-signal circuit" typically refers to the linearized model of a circuit around a bias or operating point. This approach is useful when analyzing the behavior of a circuit for small variations around a stable operating point.

Simulation1: We injected the small-signal differential circuit into the first amplifier to observe its chaotic behavior during simulation.

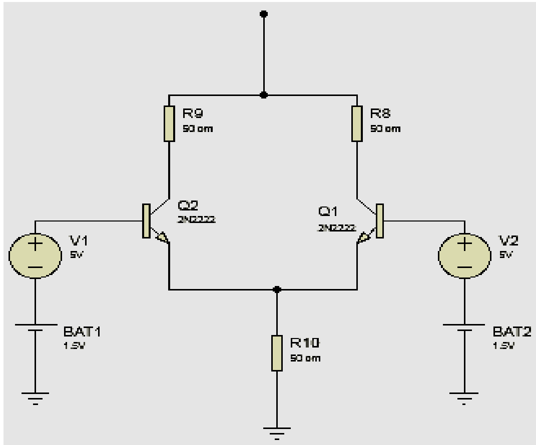
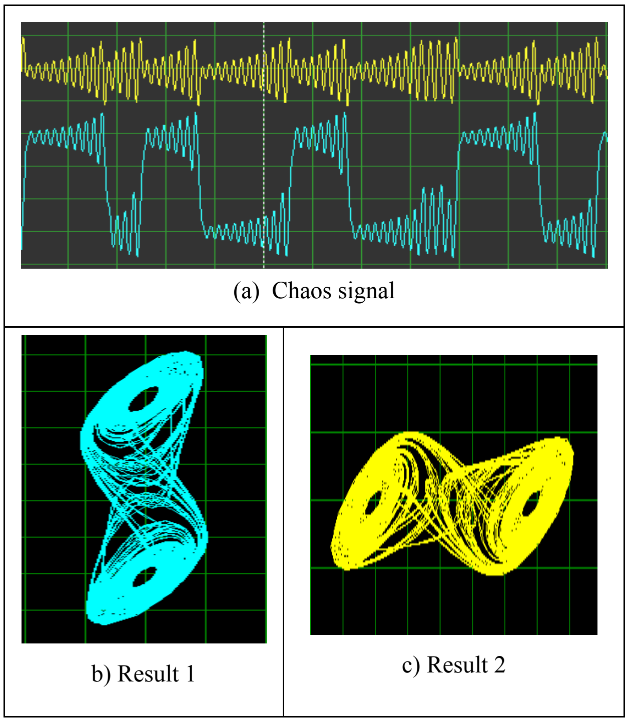


Figure 4. Small Signal Circuit

Simulation2: Figure 5 shows another three results (fig5.a , fig5.b and fig5.c).

The results are presented for the combination between chua circuit and figure4, which agree with the Proteus outputs of the chaotic system proposed as shown in Figure 5. These results confirm the physical feasibility of our proposed combination.



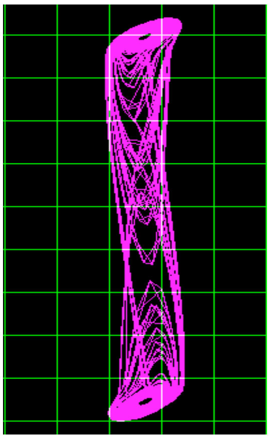
(a) Chaos signal

b) Result 1

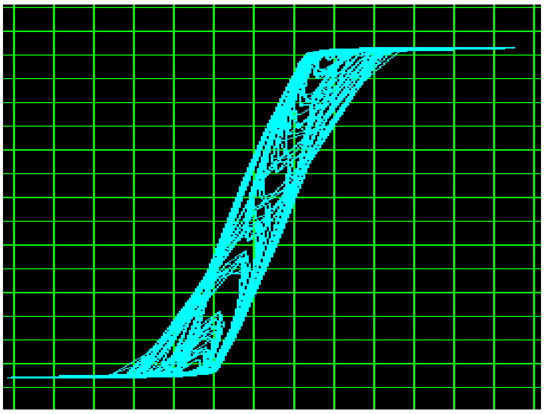
c) Result 2

Figure5. Proteus outputs of the proposed chaotic system equivalent circuit

Simulation3: We injected the small signal differential circuit between the two amplifiers into their positive terminal to see the chaos behavior in the simulation. Figure 6 shows another result chaotic system.



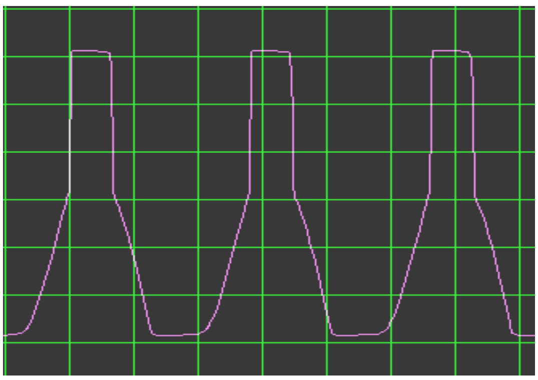
(a) Result1



(b) Result2

Figure 6. Proteus outputs of another chaotic.

Simulation4: We injected the small signal differential circuit into the negatif terminal of the second amplifiers to see the chaos behavior in the simulation. Figure 7 shows another result of the chaotic system.



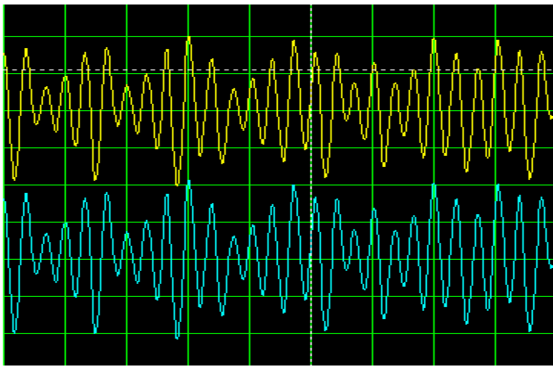
Output signal

Figure7. Proteus outputs of converge and stability chaotic systems.

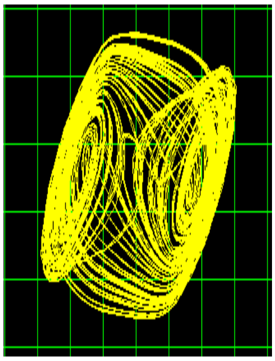
In the context of chaotic systems, "convergence" usually refers to the behavior of trajectories within the system's attractor. Chaotic systems often exhibit sensitivity to initial conditions, which means that nearby trajectories may diverge exponentially over time.

Circuit2: 2 Cascades of Two Chua Circuits:

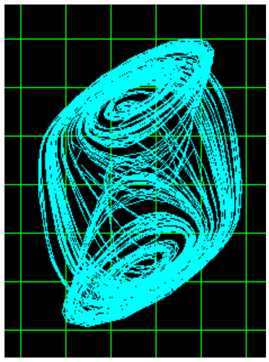
In a cascade configuration, the output of one Chua circuit serves as the input to another, forming a chain of interconnected circuits. The combined dynamics of these cascades can lead to various phenomena, including chaotic attractors, periodic orbits, and sensitivity to initial conditions. Figure 8 shows the phenomena.



(a) Output Signal



(b) Result 1



(b) Result 2

Figure 8. Proteus outputs of the proposed chaotic system equivalent circuit

The behavior of two cascades of two Chua circuits can be complex and dynamic. The Chua circuit is known for exhibiting chaotic behavior, and when two cascades of such circuits are interconnected, the system's behavior becomes even more intricate.

Circuit3: Modulation Circuit

A modulation circuit is an electronic circuit designed to modulate a signal, which means varying certain parameters of the signal to encode information or achieve a specific purpose.

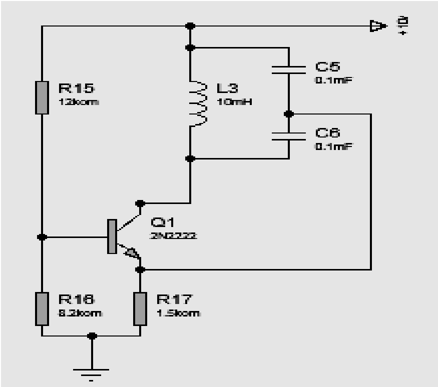
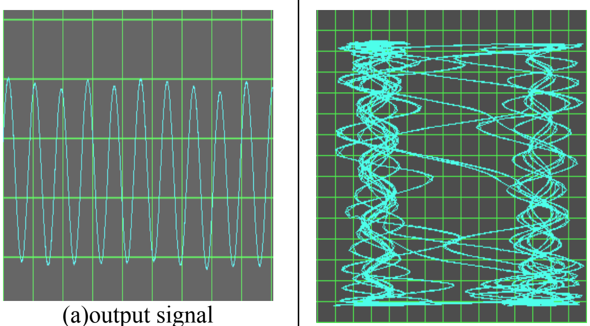


Figure 9. Modulation Circuit

Simulation: We injected the modulation circuit (Figure 9) into the first amplifier of the Chua circuit to observe the chaotic behavior in the simulation.

Figure 10 shows the result from combining chua circuit and modulation circuit.



(a)output signal

(b) result

Figure10. Proteus outputs of the combining chua and modulation circuits.

Circuit 4 : Chua circuit + Resistive Splitter Circuit

We introduced the resistive splitter circuit (Figure 11) into the first amplifier of the circuit, as illustrated in the figure below. The purpose of this setup is to explore the chaotic behavior of the system during simulation.

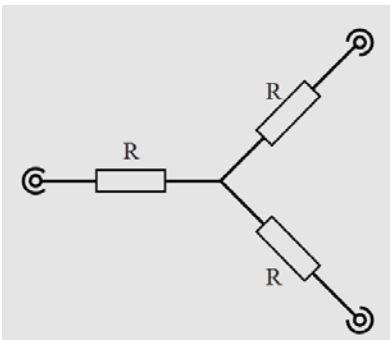
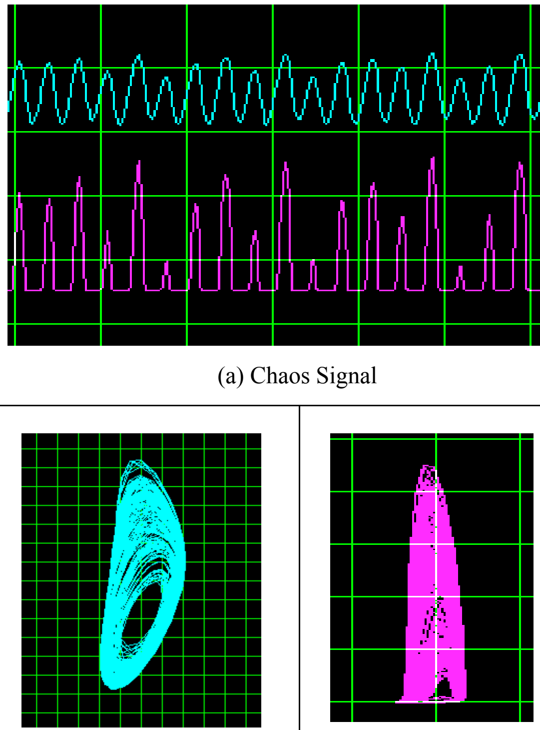


Figure 11. Resistive Splitter Circuit

Simulation: Figure 12 presents the result of the simulation.



(a) Chaos Signal

(b) Result1

(c) Result2

Figure12. Proteus outputs of chaotic systems.

B. Chua Attractor with Sierpenski Triangle Schema:

The Sierpinski Triangle is a geometric fractal named after the Polish mathematician Wacław Sierpinski. It is created through a simple iterative process. The concept, akin to the Sierpinski Triangle, inspired us to explore the integration between and among components.

Circuit1: Combining the Chua circuit with three resistors. (Sierpenski triangle schema). To create a Chua attractor with an oscillator using the Chua circuit, we add a Sierpenski Triangle (figure 13) to the Chua circuit the following figures show results that exhibit chaotic behavior.

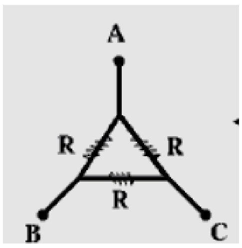
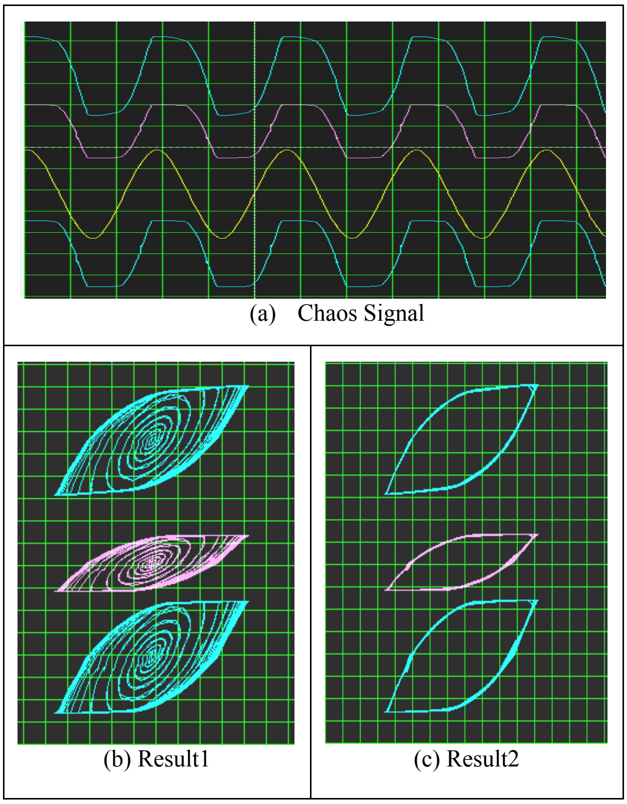


Figure13. Sierpenski Triangle (3 Resistors)

Simulation: We introduce the Sierpenski Triangle between the two amplifiers to observe the chaotic behavior during simulation. Figure 14 presents the result of the simulation.



(a) Chaos Signal

(b) Result1

(c) Result2

Figure14: Proteus outputs of converge and stability chaotic systems.

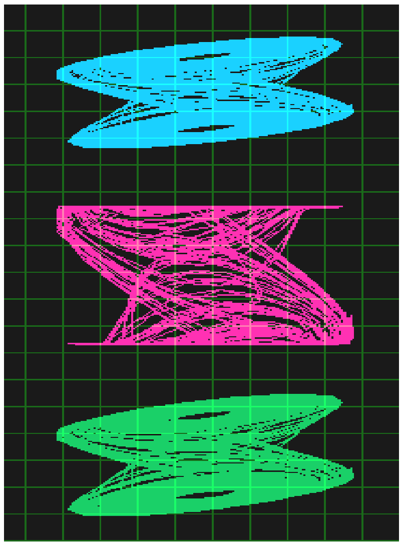
In a chaotic regime, the Chua circuit exhibits complex, non-periodic behavior. Chaotic systems are characterized by sensitivity to initial conditions, meaning small changes in the starting conditions can lead to vastly different trajectories.

Convergence (Fig14.b) to Stability (Fig14.c): Stability typically implies that a system settles into a predictable and repeatable state over time. But, chaotic systems, by definition, are not stable in the conventional sense.

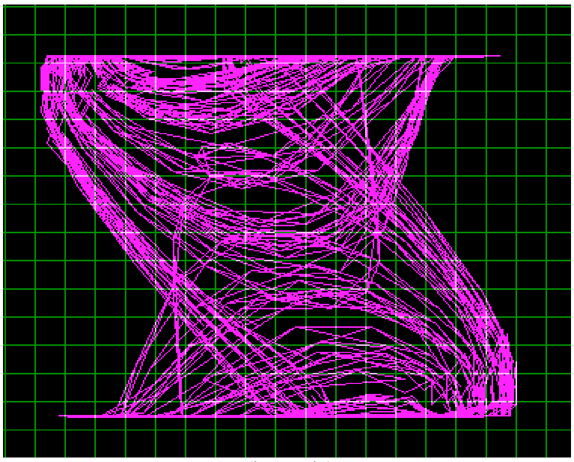
<u>Circuit 2</u>: We introduced the Sierpenski Triangle circuit into the second amplifier of the circuit, as illustrated in the figure below. The purpose of this setup is to explore the chaotic behavior of the system during simulation.

<u>Simulation:</u>

Figure 16 shows the result of the Chua circuit simulation, revealing a distinctive random dynamic with two double attractors and a new inverted attractor, as shown in Figure15-a and Figure15-b. Figure 15(b) presents the inverted attractor more explicitly.



(a) Result1



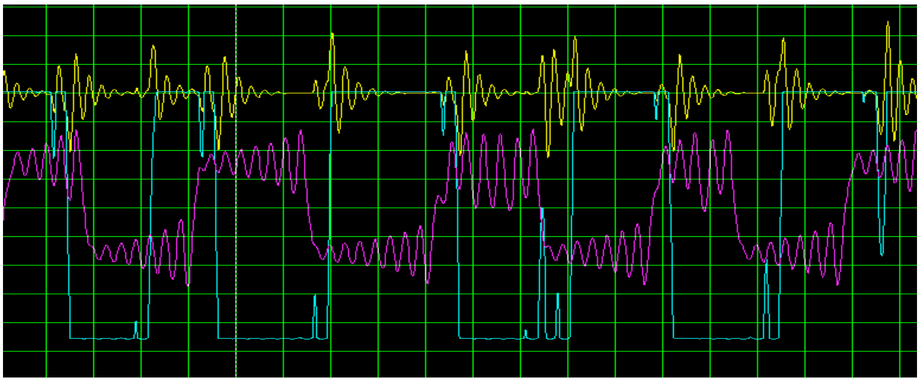
(b Result2

Figure 15. Proteus outputs of chaotic systems.

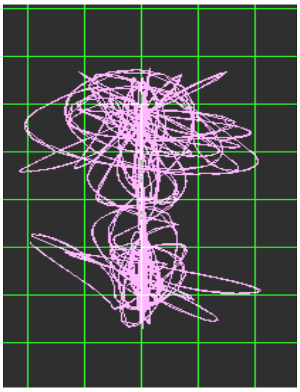
<u>Circuit 3: 3 Chua circuits (In the form of a triangle with three vertices)</u>

We replace the 3 resistors in the Sierpinski Triangle with 3 Chua circuits, and we present the results of this modification.

<u>Simulation1:</u> Figure 16 shows the first result.



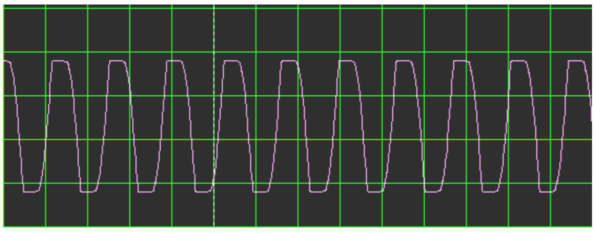
(a) Chaos signal



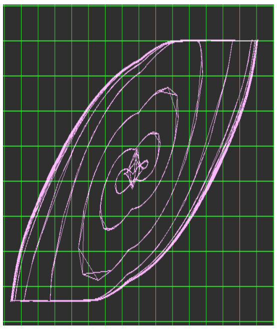
(b) Result 1

Figure 16. Proteus Output of Chaotic System

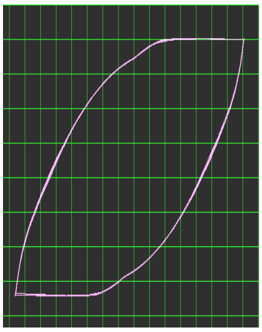
Simulation2: Figure 17 shows the second result of three Chua Circuits in form with three triangles.



(a) Chaos Signal



(b) Result1



(c) Result2

Figure17. Another result from converges and stability chaotic system.

We can represent the converge and stability chaotic system to combine three Chua circuits in the form of a triangle.

Simulation3: Figure 18 present a chaotic symmetric closed loop relative to the axis.

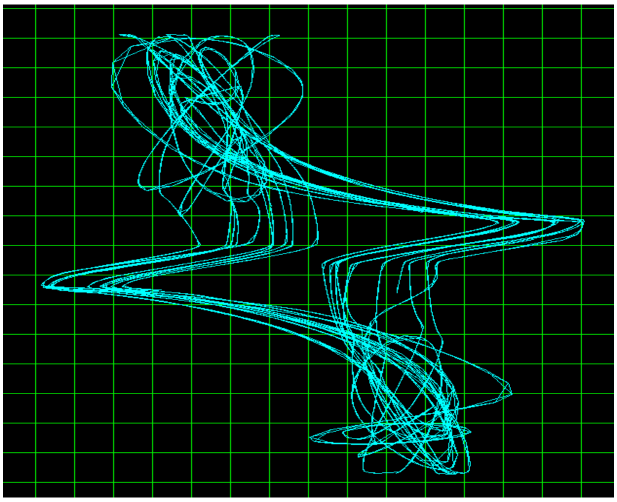


Figure18. Proteus output of chaotic system

Several factors contribute to the modification of attractors in chaotic systems over time. Chaotic systems are inherently nonlinear, and their behavior is often described by complex mathematical equations. Nonlinear dynamics can lead to a wide range of behaviors, including bifurcations and the creation of new attractors or modifications to existing ones as system parameters change.

IV. CONCLUSION

Our simulation of chaotic behavior using the Chua circuit has yielded insightful results and contributed to the understanding of complex dynamics in electronic systems. Through careful analysis, we observed the circuit's response to various input conditions, revealing intricate patterns synonymous with chaos and fractals. The Chua circuit, renowned for its chaotic signal generation capabilities, serves as a valuable tool for exploring and comprehending the complexities of dynamic systems. The findings from this study have practical implications, potentially influencing applications in signal processing.

DOI 10.5013/IJSSST.a.25.01.05

ISSN: 1473-804x online, 1473-8031 print

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